Dark energy from massive gravity redux

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Dawn of Gravitational-wave Cosmology and Theory of Gravity
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Dark energy from scalar-tensor class

- Widely used modified gravity: additional scalar ϕ.

  Potential consequences:

  - *Background:* generalises Λ to $w(t)$
  - *Scalar perturbations:* modified matter growth, non-zero anisotropic stress, ...
  - *Tensor perturbations:* non-luminal speed c_T
  - Screening of the fifth force

- Framework: **Scalar-Tensor theory class** (Horndeski + beyond Horndeski + DHOST)

  Horndeski'74, Gleyzes+'14, Langlois+'16, Ben Achour+'16

- Covers a wide variety of approaches to modelling dark energy.
- The EFT formulation is a valuable framework for interpreting data.

The purge of LIGO/Virgo

1) GW170817/GRB170817A rules out models with $c_T \neq 1$ Creminelli+'17, Ezquiaga+'17

2) Decay of GW into DE scalar via $\ddot{\gamma}_{ij}\partial_i\pi\partial_j\pi$ Creminelli+'18 '19

→ Bounds on α_H (beyond-Horndeski effects)

3) Instability induced by GW, due to term $\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi$ Creminelli+'19

→ Bounds on α_B (kinetic braiding effects)

$$\mathscr{L} = P(\phi, X) + C(\phi, X)R + \frac{6C_{,X}(\phi, X)^2}{C(\phi, X)}\phi^{;\mu}\phi_{;\mu\nu}\phi_{;\lambda}\phi^{;\nu\lambda}$$

Caveats:

- LIGO frequency band 10 – 100 Hz, while EFT cutoff ~ 100 Hz de Rham, Melville '18
- The bounds in 2) and 3) rely on sub-luminal scalar propagation $c_S \leq 1$

Going beyond scalar-tensor

- DE models in S-T class now reduced to a simple extension that includes ΛCDM as a limit. **Not falsifiable**. Do not address naturalness.
- Fate of alternative models in other modified gravity theories?

➡ *Is S-T class an accurate proxy for other theories with a scalar mode?*

- In this talk, I will revisit **massive gravity**, to obtain new alternative DE models, and to determine the extent of conclusions drawn from S-T.
- Some common properties with S-T: single scalar dof, screening mechanism, similar EFT cutoff...
- *Punchline:* Equipped with a new theory class, we can get potentially falsifiable dark energy models from massive gravity. Scalar sector is not fully described by S-T framework, although there are many common features.

dRGT Massive Gravity in a nutshell

- dRGT action for massive spin-2 field (5 gravitational dof)

de Rham,Gabadadze, Tolley '11

$$S_{\mathrm{dRGT}} = \int d^4x\sqrt{-g}\left\{\frac{M_{Pl}^2}{2}\left[R - 2\,m^2\sum_{n=0}^{4}\alpha_n\,e_n\left(\mathbb{1}-\sqrt{g^{-1}f}\right)\right] + \mathcal{L}_{\mathrm{matter}}\right\}$$

- Mass term defined using the *fiducial metric*

$$f_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\,\partial_\nu\phi^b$$

- Mass breaks diffeos. Introduced Stückelberg fields to reinstate covariance: ϕ^a. Internal Poincaré symmetry in the field space.

- Graviton potentials are given by elementary symmetric polynomials

$$e_0(X) \equiv 1\,, \qquad e_1(X) \equiv [X]\,, \qquad e_2(X) \equiv \frac{1}{2}\left([X]^2 - [X^2]\right),$$

$$e_3(X) \equiv \frac{1}{6}\left([X]^3 - 3\,[X]\,[X^2] + 2\,[X^3]\right), \qquad e_4(X) \equiv \det X$$

$$[X] \equiv X^\mu{}_\mu$$

dRGT cosmology in a nutshell

AEG, Lin, Mukohyama '11

- Only open FLRW with $K<0$ can be accommodated

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2\Omega^K_{ij}dx^i dx^j$$

Ω^K_{ij} : 3-metric of space with constant curvature K

- Field configuration:

$$\phi^0 = f(t)\sqrt{1-K\left(x^2+y^2+z^2\right)}\,, \qquad \phi^i = f(t)\sqrt{-K}\,x^i$$

- Fiducial metric is Minkowski in open chart:

$$f_{\mu\nu}dx^\mu dx^\nu = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b dx^\mu dx^\nu = -\dot{f}^2dt^2 - K\,f(t)^2\Omega^K_{ij}dx^i dx^j$$

- BG eoms:

$$3\left(H^2+\frac{K}{a^2}\right) = m^2\,L + \frac{\rho}{M^2_{Pl}}\,,$$

$$2\left(\dot{H}-\frac{K}{a^2}\right) = m^2 J\left(\tilde{c}-1\right)\xi - \frac{\rho+P}{M^2_{Pl}}\,,$$

$$\dot{\rho} = -3\,H\left(\rho+P\right),$$

$$3\left(H-\frac{\sqrt{-K}}{a}\right)J = 0$$

- Stückelberg eom forces $J=0$, thus constant ξ
- L constant → background exactly ΛCDM
- Problem: scalar and vector modes' kinetic term $\propto J$
- Strong coupling/non-linear ghost. Cannot trust perturbative expansion.

AEG, Lin, Mukohyama '12
De Felice, AEG, Mukohyama '12

$$\xi \equiv \frac{\sqrt{-K}\,f}{a}\,, \qquad \tilde{c} \equiv \frac{a\,\dot{f}}{\sqrt{-K}\,f}$$

$$L \equiv -\alpha_0 + (3\,\xi-4)\alpha_1 - 3\,(\xi-1)(\xi-2)\alpha_2 + (\xi-1)^2(\xi-4)\,\alpha_3 + (\xi-1)^3\alpha_4$$

$$J \equiv \alpha_1 + (3-2\,\xi)\alpha_2 + (\xi-1)(\xi-3)\,\alpha_3 + (\xi-1)^2\alpha_4$$

Potential resolutions

- ***Break symmetry of background***
  - approximate FLRW solutions D'Amico+'11
  - broken isotropy AEG, Lin, Mukohyama'12 ; De Felice, AEG, Lin, Mukohyama'13
  - broken homogeneity Gratia+'12
- ***Add more degrees of freedom***
  - Scalar field: quasi-dilaton, varying mass theory D'Amico+'12 ; Huang+'12
  - Bimetric: Dynamical f metric Hassan, Rosen'11
  - Multiple tensors: trimetric, multimetric.. Khosravi+'11 ; Hinterbichler, Rosen'12 ; Nomura, Soda'12 ...
- ***Break symmetry of the theory***
  - SO(3) invariant theories e.g. Comelli+'13
  - Minimal massive gravity De Felice, Mukohyama'17

In this talk, I will adopt the third approach, but in a way that preserves Lorentz invariance with 5 dof

Extending with broken translation

- We consider breaking of global $\phi^a \to \phi^a + c^a$ de Rham, Keltner, Tolley '14
  - The theory space continuous with dRGT: ***no new degrees of freedom***.
  - Lorentz invariance preserved.
  - Can use ***new invariant*** $X \equiv \eta_{ab}\phi^a\phi^b$ as a building block.
- How to find the complete class of theories:
  - *Brute force:* Determine all possible invariants, then require 5 propagating dof
  - *Shortcut:* Starting from dRGT, transform the physical and internal metrics via

$$g_{\mu\nu} \to C(X) g_{\mu\nu} \,, \quad \eta_{ab} \to \Omega(X)\eta_{ab} + D(X)\,\phi_a\phi_b$$

AEG, Kimura, Koyama'20

A new theory class for Massive Gravity

- A general class of massive gravity theories with broken shift symmetry in the field space. 6 arbitrary functions of the new invariant $X \equiv \eta_{ab}\phi^a\phi^b$

$$S = \int d^4x\sqrt{-g}\frac{M_{Pl}^2}{2}\left[C(X)\,R + \frac{3\,C'(X)^2}{2\,C(X)}\,\partial_\mu X\,\partial^\mu X - 2\,m^2\sum_{n=0}^{4}\alpha_n(X)\,e_n\left(\mathbb{1} - \sqrt{g^{-1}\tilde{f}}\right)\right]$$

where $\tilde{f}_{\mu\nu} \equiv (\eta_{ab} + D(X)\,\phi_a\phi_b)\partial_\mu\phi^a\,\partial_\nu\phi^b$

AEG, Kimura, Koyama'20

Examples:

- dRGT – $C = 1\,,\ D = 0\,,\ \alpha_n = \text{constant}$
- Generalised Massive Gravity (GMG) – $C = 1\,,\ D = 0$ de Rham, Fasiello, Tolley '14

Cosmology in GMG

- In the rest of the talk, I focus on GMG:

$$S_{GMG} = \int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} \left[R - 2\,m^2 \sum_{n=0}^{4} \alpha_n(X)\, e_n\left(\sqrt{\mathbb{1} - g^{-1}f}\right) \right] + \mathcal{L}_{\rm matter} \right\}$$

- Metric ansatz:

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2 \Omega_{ij}^K dx^i dx^j$$

Ω_{ij}^K : 3-metric of space with constant curvature K

- Homogeneity/isotropy:
  - $f_{\mu\nu}$ should have the same FLRW form as $g_{\mu\nu}$
  - $X \equiv \eta_{ab}\phi^a\phi^b$ should be uniform.
- Only open universe $K < 0$ allowed, with

$$\phi^0 = f(t)\sqrt{1 - K\left(x^2 + y^2 + z^2\right)}\,, \qquad \phi^i = f(t)\sqrt{-K}\,x^i \qquad \Longrightarrow \quad \phi^a\phi_a = -f(t)^2$$

$$f_{\mu\nu}dx^\mu dx^\nu \quad = \quad \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b dx^\mu dx^\nu = -\dot{f}^2 dt^2 - K\,f(t)^2\Omega_{ij}^K dx^i dx^j$$

Background dynamics in GMG

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2\Omega^K_{ij}dx^i dx^j$$
$$f_{\mu\nu}dx^\mu dx^\nu = -\dot{f}^2 dt^2 - K\, f(t)^2\Omega^K_{ij}dx^i dx^j$$
$$\phi^a\phi_a = -f(t)^2$$

- Background equations of motion:

$$3\left(H^2 + \frac{K}{a^2}\right) = m^2\, L + \frac{\rho}{M_{Pl}^2}\,,$$
$$2\left(\dot{H} - \frac{K}{a^2}\right) = m^2 J\,(\tilde{c} - 1)\,\xi - \frac{\rho + P}{M_{Pl}^2}\,,$$
$$\dot{\rho} = -3\,H\,(\rho + P)\,,$$

Same as in dRGT

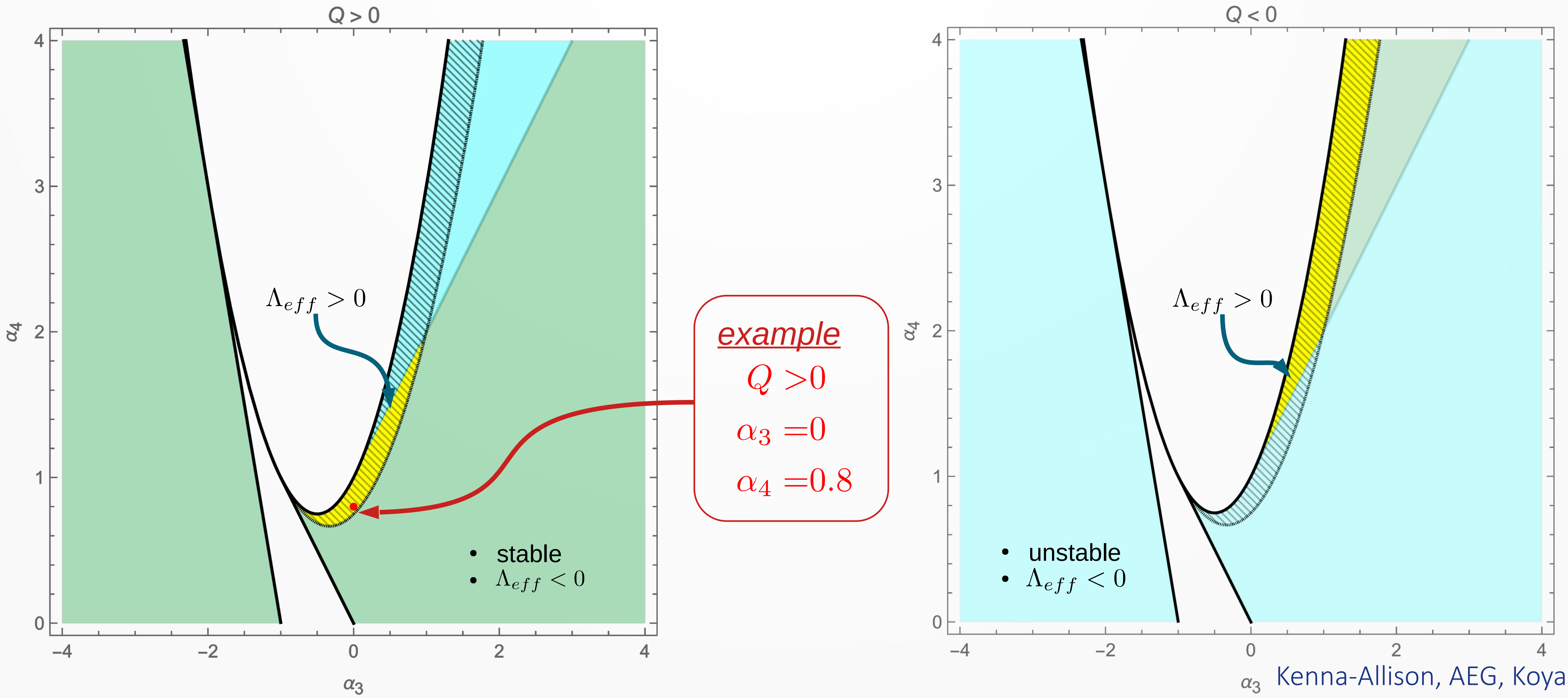
$$3\left(H - \frac{\sqrt{-K}}{a}\right) J = -\frac{2\,a\,\xi}{\sqrt{-K}}\,\partial_X L$$

In dRGT $J = 0$

$$\xi \equiv \frac{\sqrt{-K}\,f}{a}\,, \quad \tilde{c} \equiv \frac{a\,\dot{f}}{\sqrt{-K}\,f}$$
$$L \equiv -\,\alpha_0 + (3\,\xi - 4)\alpha_1 - 3\,(\xi - 1)(\xi - 2)\alpha_2 + (\xi - 1)^2(\xi - 4)\,\alpha_3 + (\xi - 1)^3\alpha_4$$
$$J \equiv \alpha_1 + (3 - 2\,\xi)\alpha_2 + (\xi - 1)(\xi - 3)\,\alpha_3 + (\xi - 1)^2\alpha_4$$
$$\partial_X L \equiv -\,\alpha_0' + (3\,\xi - 4)\alpha_1' - 3\,(\xi - 1)(\xi - 2)\alpha_2' + (\xi - 1)^2(\xi - 4)\,\alpha_3' + (\xi - 1)^3\alpha_4'$$

Perturbative stability

- Minimal model: $\alpha_0 = \alpha_1 = 0\,,\ \alpha_2 = 1 + (10^{-4} Q\, H_0^2)\, \phi^a \phi_a\,,\ \alpha_3, \alpha_4 = \text{constant}$



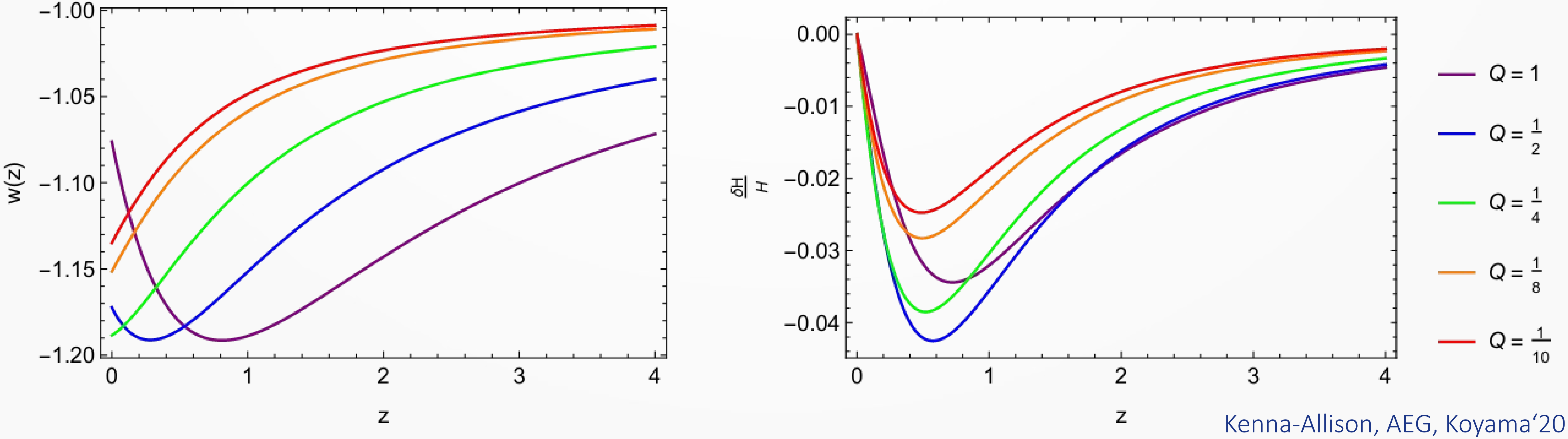
Kenna-Allison, AEG, Koyama'19

Background cosmology

- Fixing the parameters:

$$\alpha_0 = \alpha_1 = 0\,,\ \alpha_2 = 1 + \left(10^{-4}\,Q\,H_0^2\right)\phi^a\phi_a\,,\ \alpha_3 = 0\,,\ \alpha_4 = 0.8\,,\ \Omega_m = 0.3\,,\ \Omega_K = 3\times\ 10^{-3}$$

- Value of m fixed such that $H = H_0$ today (typically $m \sim \mathcal{O}(1)\,H_0$)
- Phantom effective fluid with $w < -1$.

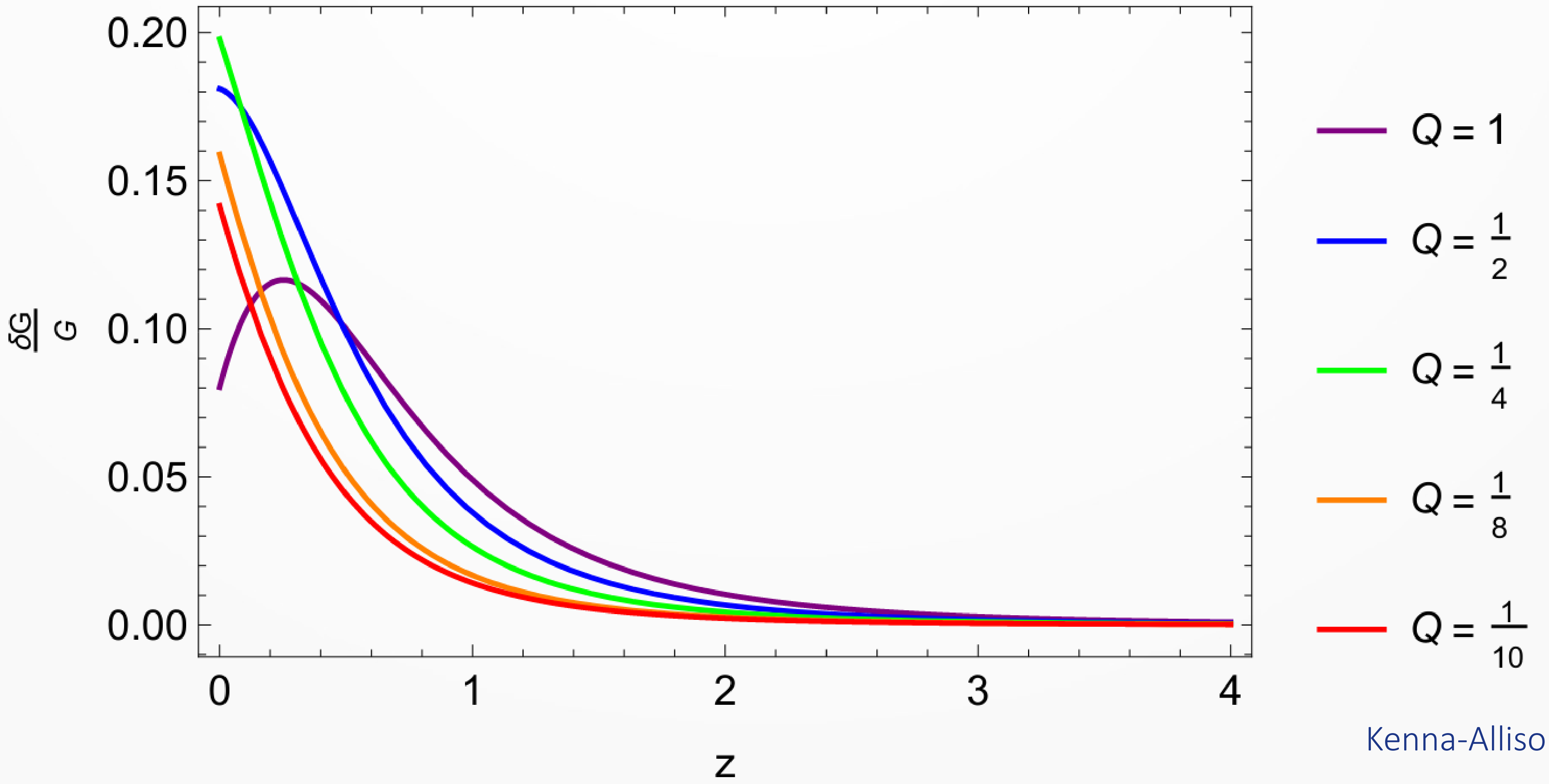


Kenna-Allison, AEG, Koyama'20

Modified gravitational constant

- Scalar graviton contributes to gravitational interactions. Effective G larger than GR value (G_N) by $\sim 10\%$

$$\frac{k^2}{a^2}\Phi = -G_{\rm eff}(z)\,\rho\,\delta_m$$

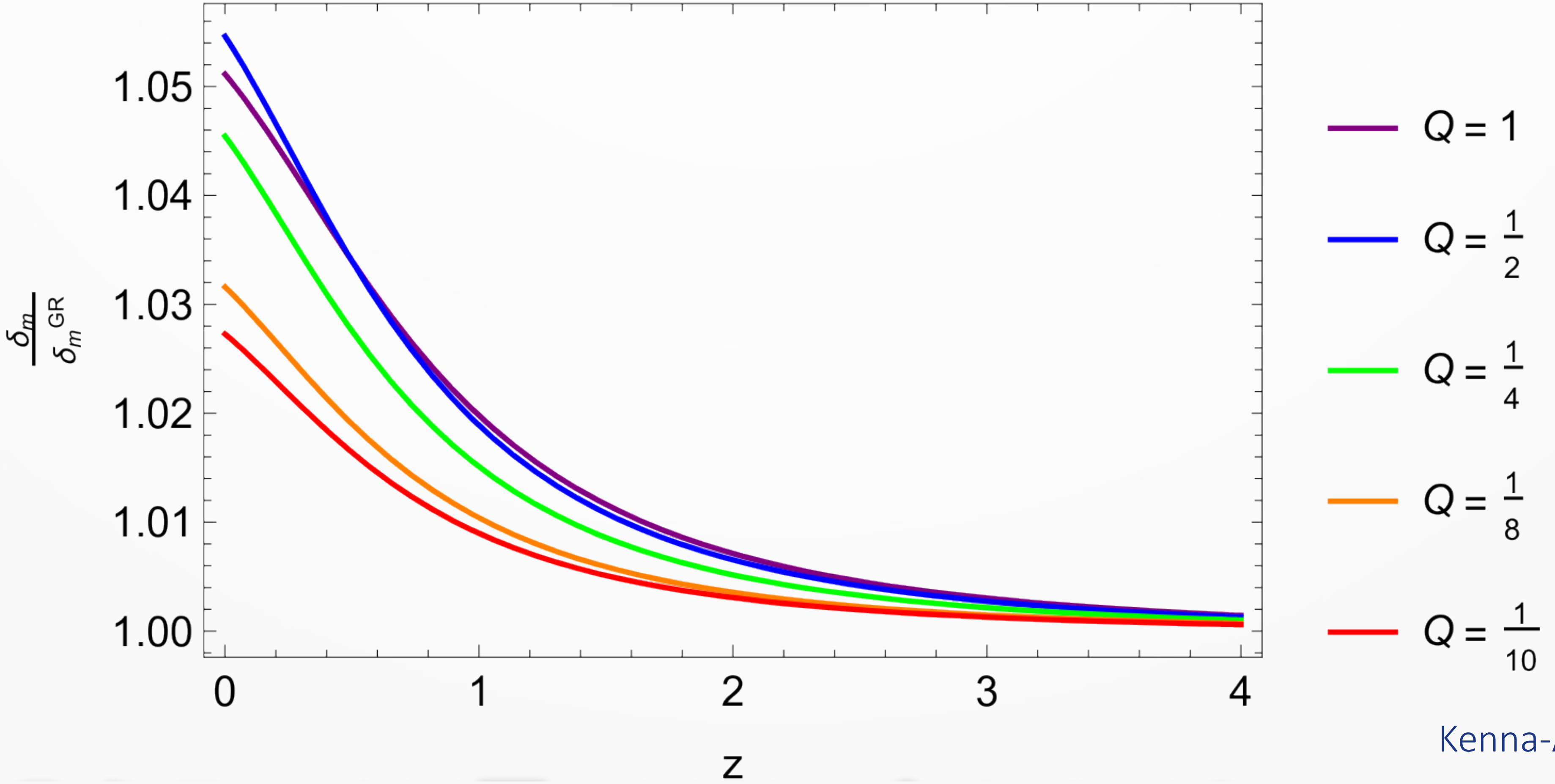


Kenna-Allison, AEG, Koyama'20

Matter growth

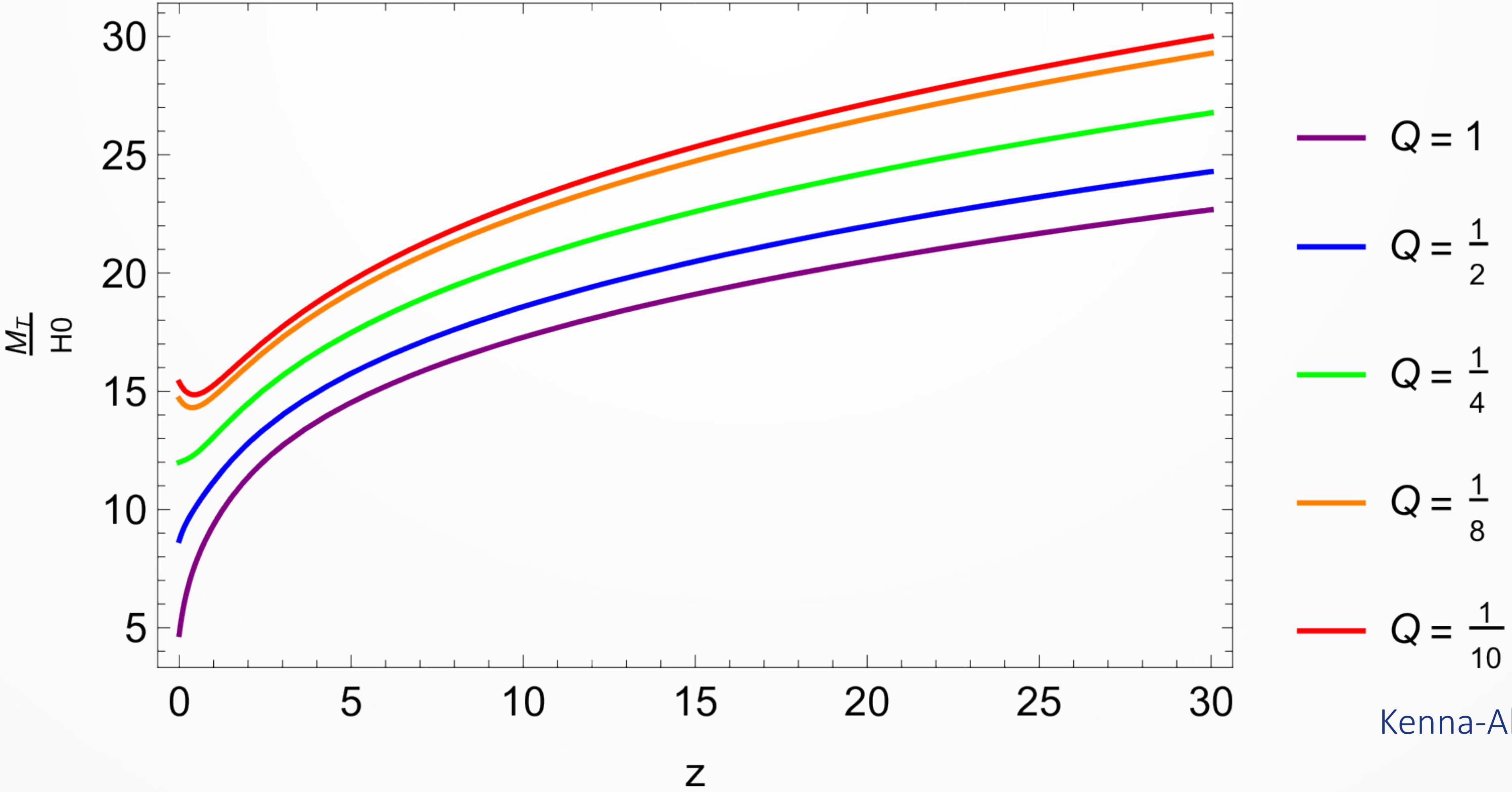
- Modification in the expansion and the Newton's constant both contribute to matter growth

$$\partial_z^2 \delta_m + \left(\frac{H'}{H} - \frac{1}{1+z} \right) \partial_z \delta_m - \frac{G_{\rm eff}\, \rho}{(1+z)^2 H^2} \delta_m = 0$$



Kenna-Allison, AEG, Koyama'20

Massive gravitational waves



Kenna-Allison, AEG, Koyama'20

- Tensor modes see the usual light cone with $c_T = 1$
- Slight modification to the speed of GW from tensor mass $M_T \sim \mathcal{O}(10)\, H_0 \sim \mathcal{O}(10^{-32})\,\mathrm{eV}$
- Well within the bounds from LIGO $M_T \leq 7.7 \times 10^{-23}\,\mathrm{eV}$ Abbott+'17

Screening the fifth force

AEG, Kimura, Kenna-Allison, Koyama'21

- Scalar dof contributes to gravity → Vainshtein screening at non-linear scales
- Non-linear scalar perturbations around FLRW (in Newtonian gauge)

$$ds^2 = -(1+2\,\Phi)dt^2 + a^2(1-2\,\Psi)(dr^2 + r^2 d\Omega_2^2)$$

- Stückelberg perturbations: $\delta\phi^a = \left(\delta\phi^0, \partial^i\Pi\right)$
- Spherically symmetric sources $\delta\rho(r)$

> - Π analogous to scalar in S-T
> - $\delta\phi^0$ has no analogue

- Non-linear expansion for short distances $H_0\, r \propto \mathcal{O}(\epsilon)$ with:

$$\delta\rho \propto \mathcal{O}(\epsilon^0)\,, \qquad \text{other perturbations } \propto \mathcal{O}(\epsilon^2)\,, \qquad \partial_r \propto \mathcal{O}(\epsilon^{-1})$$

- Gravitational potentials obtained from Einstein's equations:

$$\frac{\Psi'}{r} = \frac{G_N\,\delta M}{r^3 a} + \frac{m^2 a^2}{2}\left(\mathcal{C}_1\frac{\Pi'}{r} + \frac{\mathcal{C}_2}{2}\frac{(\Pi')^2}{r^2} + \frac{\mathcal{C}_3}{3}\frac{(\Pi')^3}{r^3}\right)$$

$$\frac{\Phi'}{r} = \frac{G_N\,\delta M}{r^3 a} - \frac{m^2 a^2}{2}\left([\mathcal{C}_1 + \mathcal{C}_2(\tilde{c}-1)]\frac{\Pi'}{r} + \mathcal{C}_3(\tilde{c}-1)\frac{(\Pi')^2}{r^2} - \frac{\mathcal{C}_3}{3}\frac{(\Pi')^3}{r^3}\right)$$

- Einstein's equations similar to Horndeski ones. But after integrating out $\delta\phi^0$, unique structure of untruncated NL interactions in Stückelberg equation.

Non-linear behaviour

AEG, Kimura, Kenna-Allison, Koyama'21

- Master equation for Π' – 9th order polynomial

$$\frac{\Phi'}{r}\left(\mathcal{C}_1+\mathcal{C}_2\frac{\Pi'}{r}+\mathcal{C}_3\frac{(\Pi')^2}{r^2}\right)-\frac{\Psi'}{r}\left(\mathcal{C}_4+\mathcal{C}_5\frac{\Pi'}{r}\right)+\frac{\sum_{i=0}^{7}\mathcal{D}_i\left(\frac{\Pi'}{r}\right)^i}{\left(\mathcal{C}_1+\mathcal{C}_2\frac{\Pi'}{r}+\mathcal{C}_3\frac{(\Pi')^2}{r^2}\right)^2}=0$$

- Linear (perturbative) solution

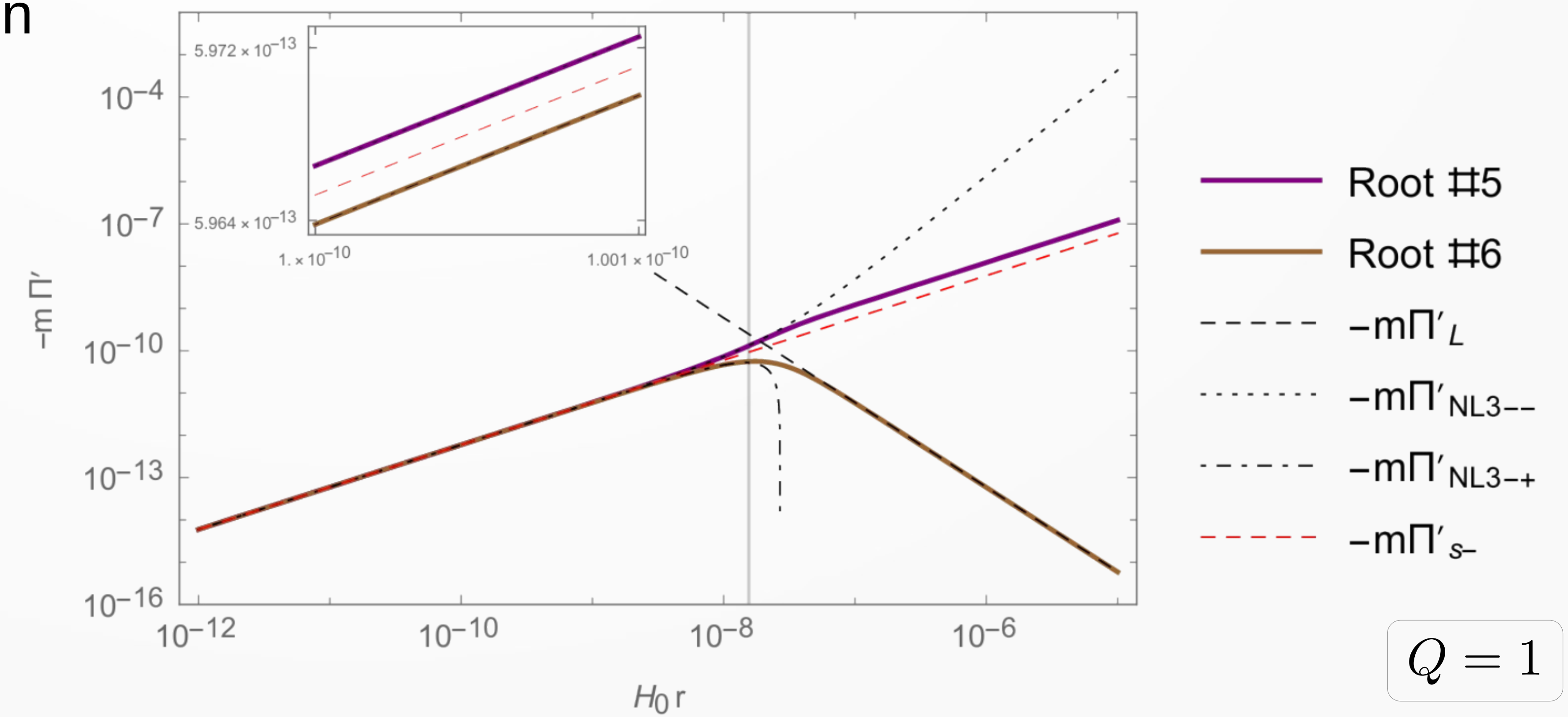
$$\Pi'_L \propto -\frac{G_N\,\delta M}{r^2}$$

- Non-linear solution:

$$\Pi'_{NL} \propto -r$$

- Vainshtein radius:

$$r_V \simeq \left(\frac{G_N\,\delta M}{m^2}\right)^{1/3}$$

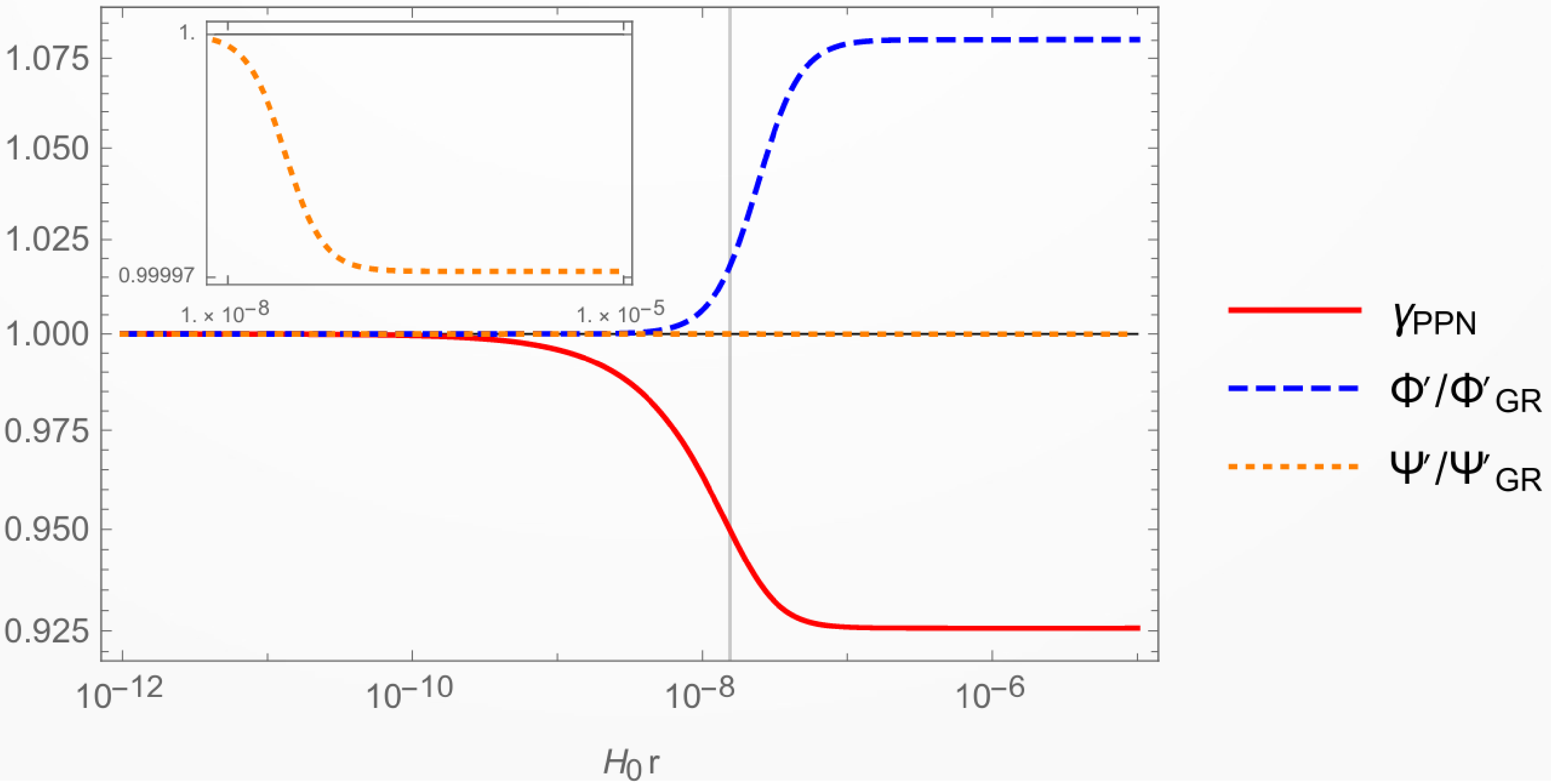


$Q=1$

Vainshtein screening

AEG, Kimura, Kenna-Allison, Koyama'21

- Vainshtein radius for the sun, $\delta M = M_{\odot}$ ⇨ $r_V \simeq 100\,\mathrm{pc}$
- PPN parameter $\gamma \equiv \Psi/\Phi$ within the Vainshtein radius $1-\gamma \propto r^3$
- Solar system tests $|1-\gamma| < 2.3\times 10^{-5}$ satisfied at ~ 0.1 pc

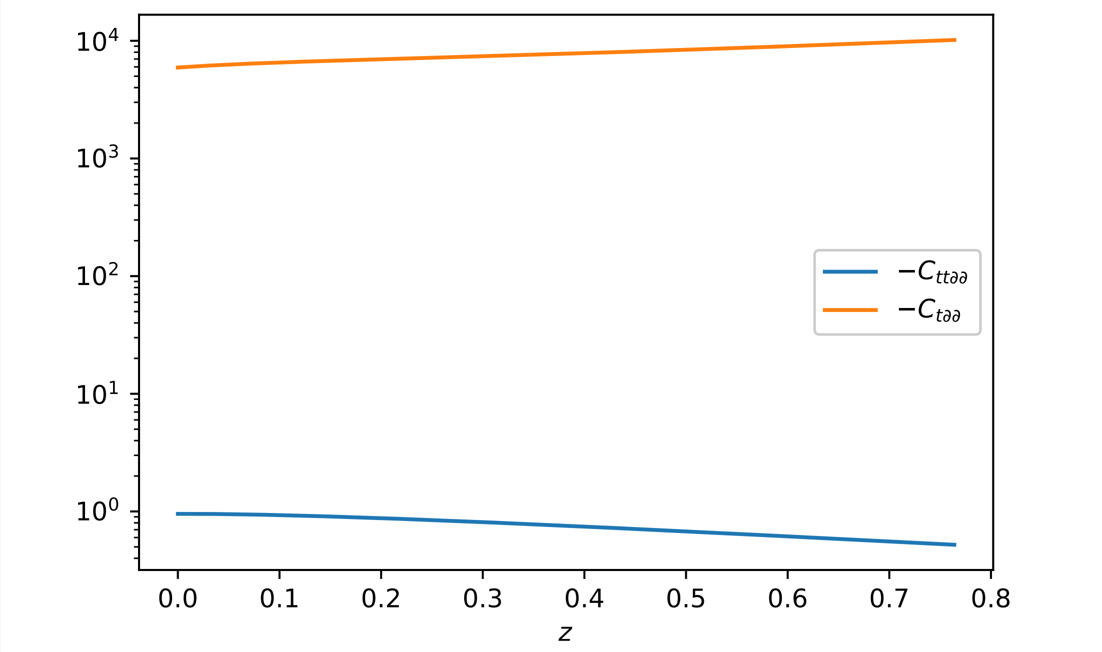


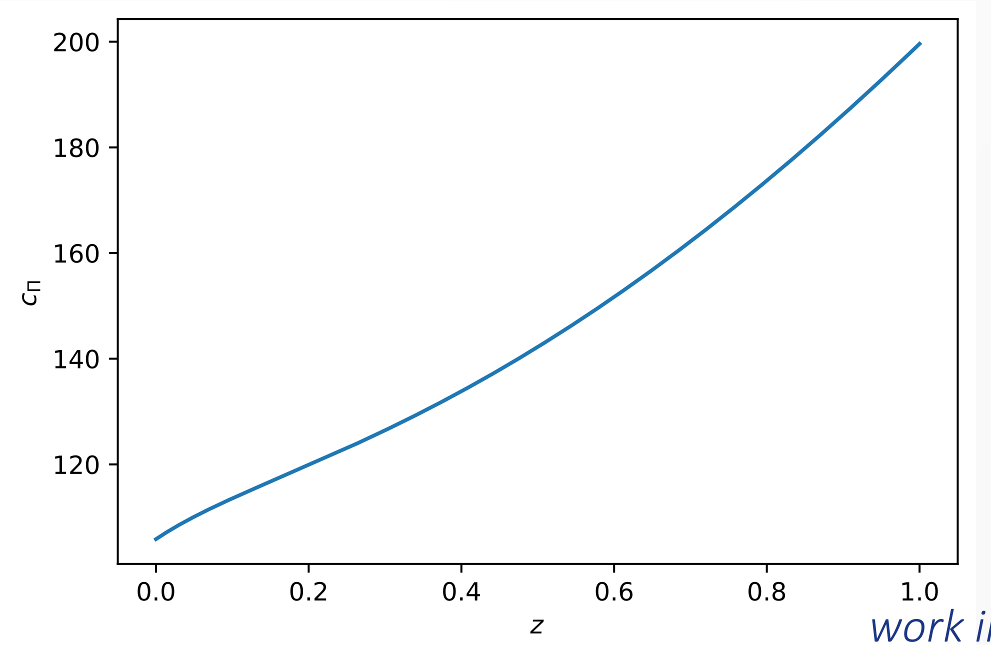
Tensor-Scalar-Scalar interactions

- How about effects due to $\ddot{\gamma}_{ij}\partial_i\pi\partial_j\pi$ and $\dot{\gamma}_{ij}\partial_i\pi\partial_j\pi$ interactions? In S-T, these arise from covariantisation (e.g. $(\nabla\phi).(\nabla\nabla\phi).(\nabla\phi) \ni \dot{\gamma}_{ij}\partial_i\pi\partial_j\pi$). How about in massive gravity?

$$\mathcal{L}_{h\Pi\Pi} = \left(\frac{C_{\partial\partial}}{M_p}\hat{\gamma}_{ij} + \frac{C_{t\partial\partial}}{\Lambda_2^2}\dot{\hat{\gamma}}_{ij} + \frac{C_{tt\partial\partial}}{\Lambda_3^3}\ddot{\hat{\gamma}}_{ij}\right)\frac{\partial^i\hat{\Pi}\,\partial^j\hat{\Pi}}{a^2} + \frac{C_{\partial\partial\partial\partial}}{\Lambda_3^3}\hat{\gamma}_{ij}\frac{\partial^i\partial^j\hat{\Pi}\,\partial^2\hat{\Pi} - \partial^i\partial^k\hat{\Pi}\,\partial_k\partial^j\hat{\Pi}}{a^4}$$

- Both are present! Feature of Λ_3 theories with Vainshtein mechanism?
- ***For the minimal example:*** coefficients $> \mathcal{O}(1)$, but $c_s > 1$ due to proximity to dRGT.





work in progress

Conclusions

- A new class of Lorentz invariant theories with 5 dof, that extend dRGT massive gravity.
- We considered a proof-of-principle cosmological model with following properties
  - ✓ perturbatively stable
  - ✓ background with an effective phantom DE
  - ✓ modified matter growth
  - ✓ massive tensor modes within LIGO bound
  - ✓ successful screening
  - ✓ distinguishable from ΛCDM and other S-T models, falsifiable.
  - ✓ smallness of c.c. technically natural
- Although massive gravity is not accurately described by the Scalar-Tensor framework, some common conclusions: Vainshtein radius, presence of scalar-tensor-tensor interactions..
- We only focussed on a small corner of the theory class. Many avenues remain unexplored (e.g. non-minimal coupling)