A Preliminary CMB Heat Flux map

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Abstract

Estimating heat flux at Core-Mantle Boundary(CMB) is very important for understanding mantle thermal convection. Thanks to the discovery of perovskite post-perovskite transition in the deep mantle, using Clapeyron slope has a possibility that can constrain thermal gradient and geotherm above CMB. Seismic data is very useful in estimating geotherm, but still have a lot of uncertainties, such as effect of composition heterogeneity in the lower mantle, spin transition of iron, radioactive heat, partial melt effect and thermal conductivity in high pressure condition.

Using the maximum heat flux assumed from the hypothesis 'Double-crossing'(Hernlund,2010) and P-wave velocity map which is highly effected in temperature, we can simply estimate the heat flux near the CMB. This method is useful in its simpleness, but the problem is assuming linear correlation between P-wave velocity and temperature.

Problems

- Uncertainties in analytical solution

Temperature distribution:

$$T = T_{CMB} - \Delta T erf\left(\frac{z}{\delta(x,t)}\right) \quad (\Delta T = T_{CMB} - T_{Lm})\ (1)$$

CMB heat flux:

$$q_{CMB} = \frac{2k\Delta T}{\delta\sqrt{\pi}}\ (2)$$

Conservation of energy:

$$\rho c_p\left(\frac{\partial T}{\partial t} + v_z\frac{\partial T}{\partial z}\right) = \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) - T\Delta s\Gamma - \rho g\alpha T v_z + Q + \theta\ (3)$$

Thermal boundary layer thickness:

$$\delta^2(t) = \frac{\delta^2(t_0) + 4\kappa\int_{t_0}^{t}\exp\left(2\int_{t_0}^{t''}\dot{\varepsilon}(t')dt'\right)dt''}{\exp\left(2\int_{t_0}^{t}\dot{\varepsilon}(t')dt'\right)}\ (4)$$

$$\delta(t\to\infty) = \sqrt{\frac{2\kappa}{\dot{\varepsilon}}}\ (5)$$

- The lower cross of the 'Double crossing' can be too thin

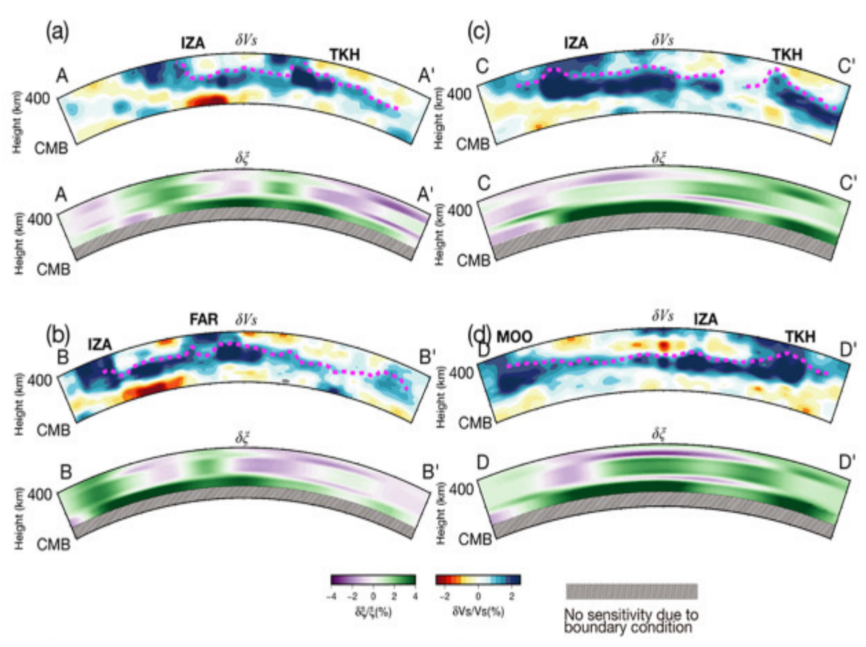


Figure1. Shear wave velocity anomaly in north pacific(Y. Suzuki et al.,2021)

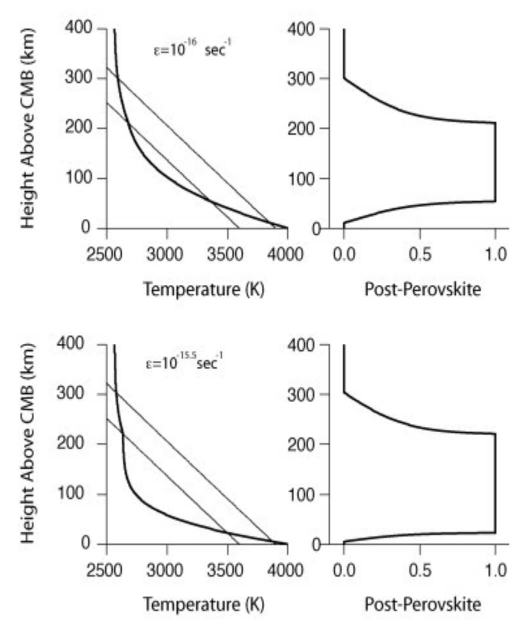


Figure2. Numerical solution of Geotherm double cross(Hernlund,2010)

Approach

- Numerical solution method:

Using steady equation of conservation of energy(6) and 'lever rule'(7);exhibits conservation of solute species, mass conservation(8,9)

$$\frac{\partial}{\partial z}\left(\rho c_p T v_z - k\frac{\partial T}{\partial z}\right) = -T\Delta s\Gamma - \rho g\alpha T v_z (6)$$

$$\frac{X - X_{pPv}}{X_{Pv} - X_{pPv}} = \phi\frac{\rho_{Pv}}{\rho_{av}}\ (7)$$

$$\frac{\partial(\rho_{Pv}\phi v_z)}{\partial z} = \Gamma\ (8)$$

$$\frac{\partial(\rho_{pPv}(1-\phi)v_z)}{\partial z} = -\Gamma\ (9)$$

Define Transition zone by binary loop:

$$T_{up} = \frac{\Delta T_{inc}}{2}\frac{X_{pPv}(1 - X_{pPv})}{X_0(1 - X_0)} + T_0 - X_{pPv}(T_0 - T_1)(10)$$

$$T_{down} = -\frac{\Delta T_{inc}}{2}\frac{X_{Pv}(1 - X_{Pv})}{X_0(1 - X_0)} + T_0 - X_{Pv}(T_0 - T_1)(11)$$

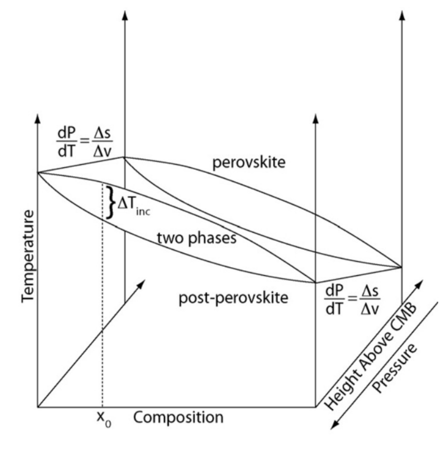


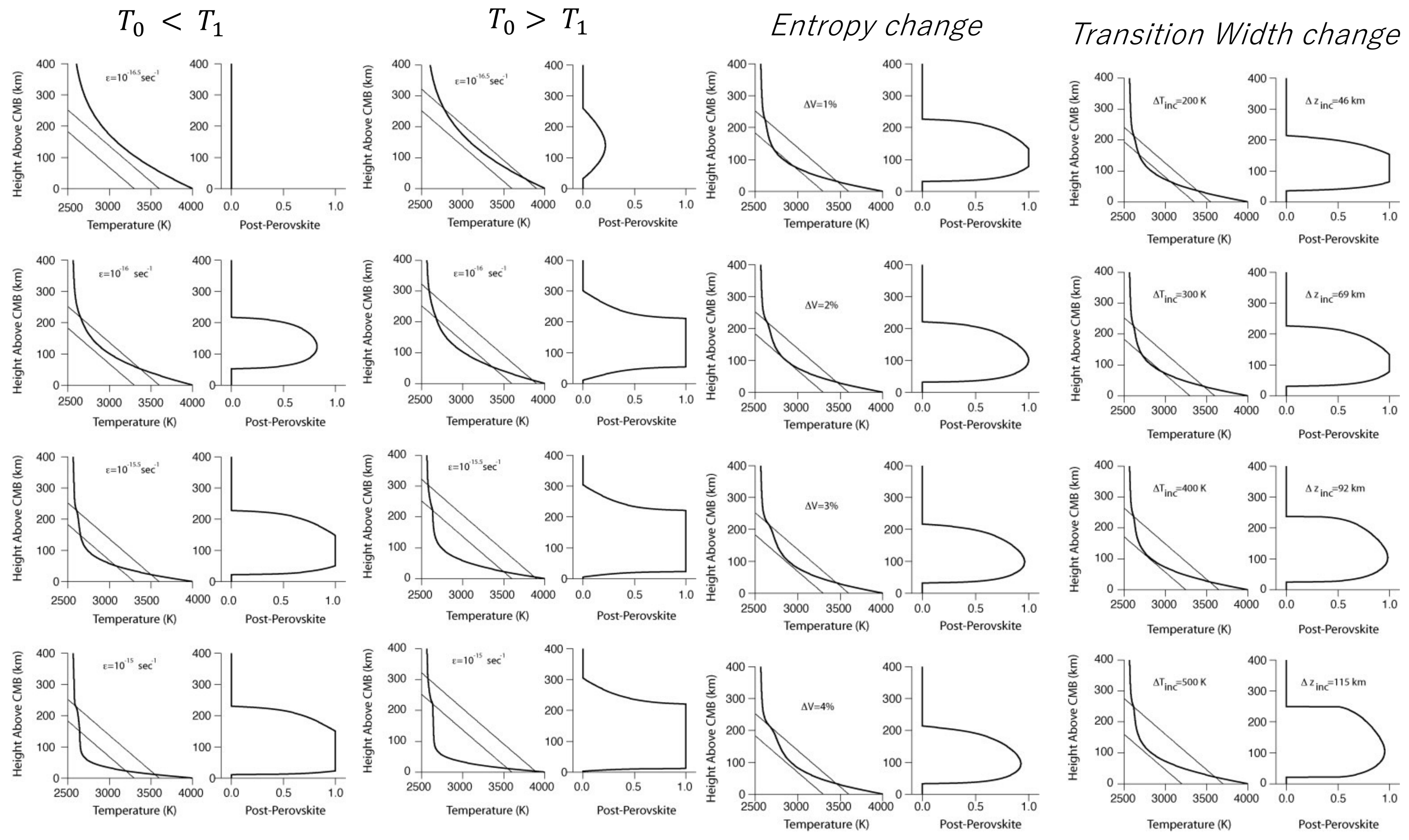
Figure3. Binary phase loop ($T_0 > T_1$)

Table 1
Parameter values considered in this study.

Quantity	Value (range, if applicable)
ρ_{Pv}	5500 kg/m^3
g	10.3 m/s^2
γ	13 MPa/K
ΔV	1 (1–4)%
ΔT_{inc}	300 (200–500) K
$\dot{\varepsilon}$	2×10^{-16} ($10^{-16.5}$–10^{-15}) sec^{-1}
T_{cmb}	4000 K
T_∞	2500 K
k_{cmb}	5 W/m/K
c_p	1200 J/K/kg
X_0	0.15
$T_{0,cmb}$	3600,3600 K
$T_{1,cmb}$	2600,4600 K

Preliminary Results

Figure4. Geotherm line and transition zone used in Hernlund(2010)



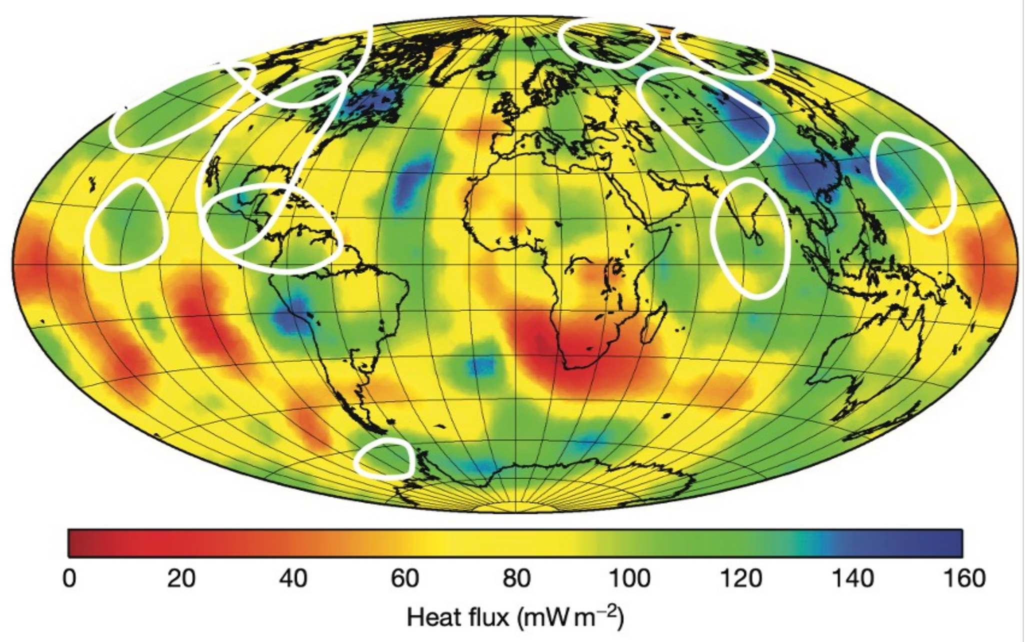


Figure5. Using heat flux range of 0-160mWm^{-2}, P-wave velocity model(Houser et al.,2008). (Treatise,2012)

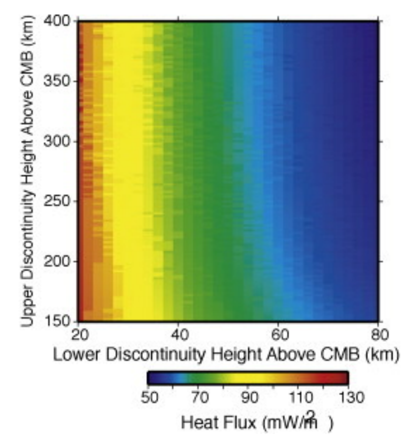


Figure6. Estimated Heat Flux from the numerical solution above. The parameters is the height of double cross.

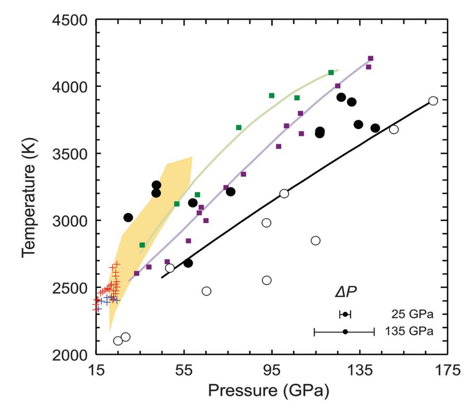


Figure7. Solidus line of Pyrolitic lower mantle. (Nomura et al.,2014)

Conclusions

- Using the numerical solution, we can directly get the temperature distribution near the CMB, where Pv-pPv transition is there
  → Assumption: D" discontinuity place is the maximum CMB heat flux

- Just assume linear correlation of temperature anomaly and P-wave velocity anomaly, we can have the heat flux map(Figure5)

$$dlnV_p \approx \left(\frac{\partial lnV_p}{\partial T}\right)\delta T\ (12)$$

Future Directions

- Put time dependent Dynamical models
  - make δ time dependent (Wu et al.,2011)
  - fix strain-rate by rheological theory

- Change the parameters
  - thermal conductivity, CMB temperature

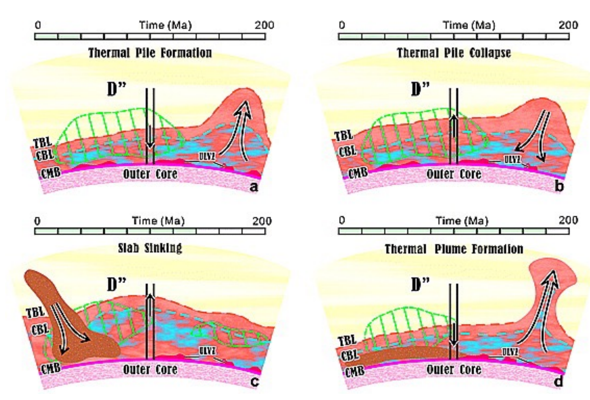


Figure8. Time dependent Thermal Boundary Layer used in Wu(2011)

- Fit with the seismic observations
  - S-wave velocity anomaly highly effected by other factors:

$$dlnV_s = \frac{\partial lnV_s}{\partial T}\delta T + \frac{\partial lnV_s}{\partial X_{Fe}}\delta X_{Fe} + \frac{\partial lnV_s}{\partial\varphi_{ppv}}\delta\varphi_{ppv}\ (13)$$

  - get rid of P-wave anomaly by other factors: using vote map?
  - test model results with velocity anomaly zones

- Change the heat flux range; make minimum

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