Quantification of the uncertainty of NDE

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Contents

1. Background and motivation

We need to accept:

- Limited resource
- Not parts but a whole
- No 100% safety

⇨ Risk-based management

1. Background and motivation

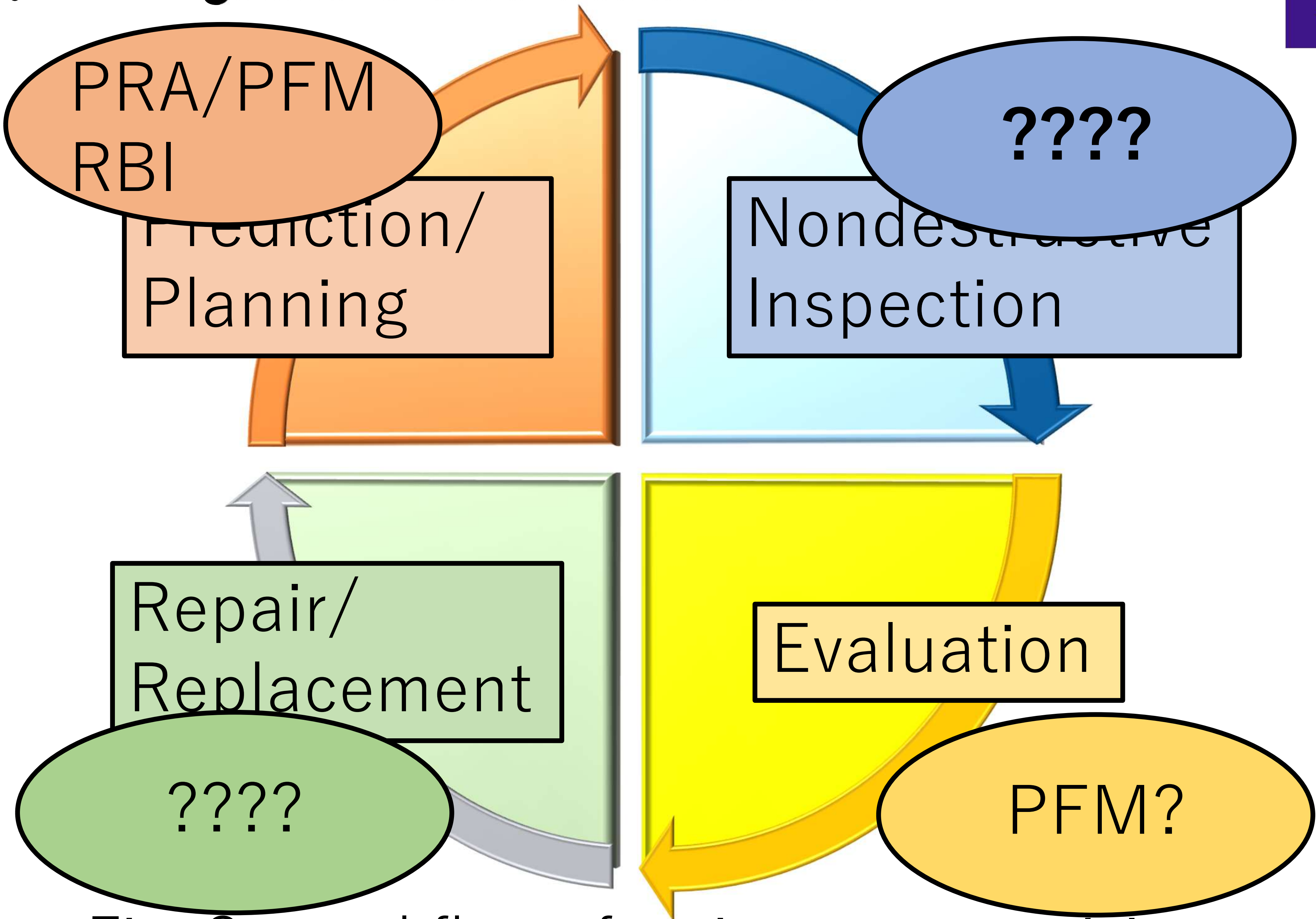


Fig. General flow of maintenance activity

1. Background and motivation

The role of nondestructive inspection

detection & evaluation

of a flaw

to estimate the size of a detected flaw

to confirm whether there is a flaw or not

1. Background and motivation

To account for the uncertainty of NDT

detection & evaluation

"safety factor"

- true size = estimation $\pm \alpha$

"minimum detectable flaw size"

- a flaw larger than a certain size->detectable
- a flaw smaller than a certain size->undetectable

Incompatible with "Risk."

1. Background and motivation

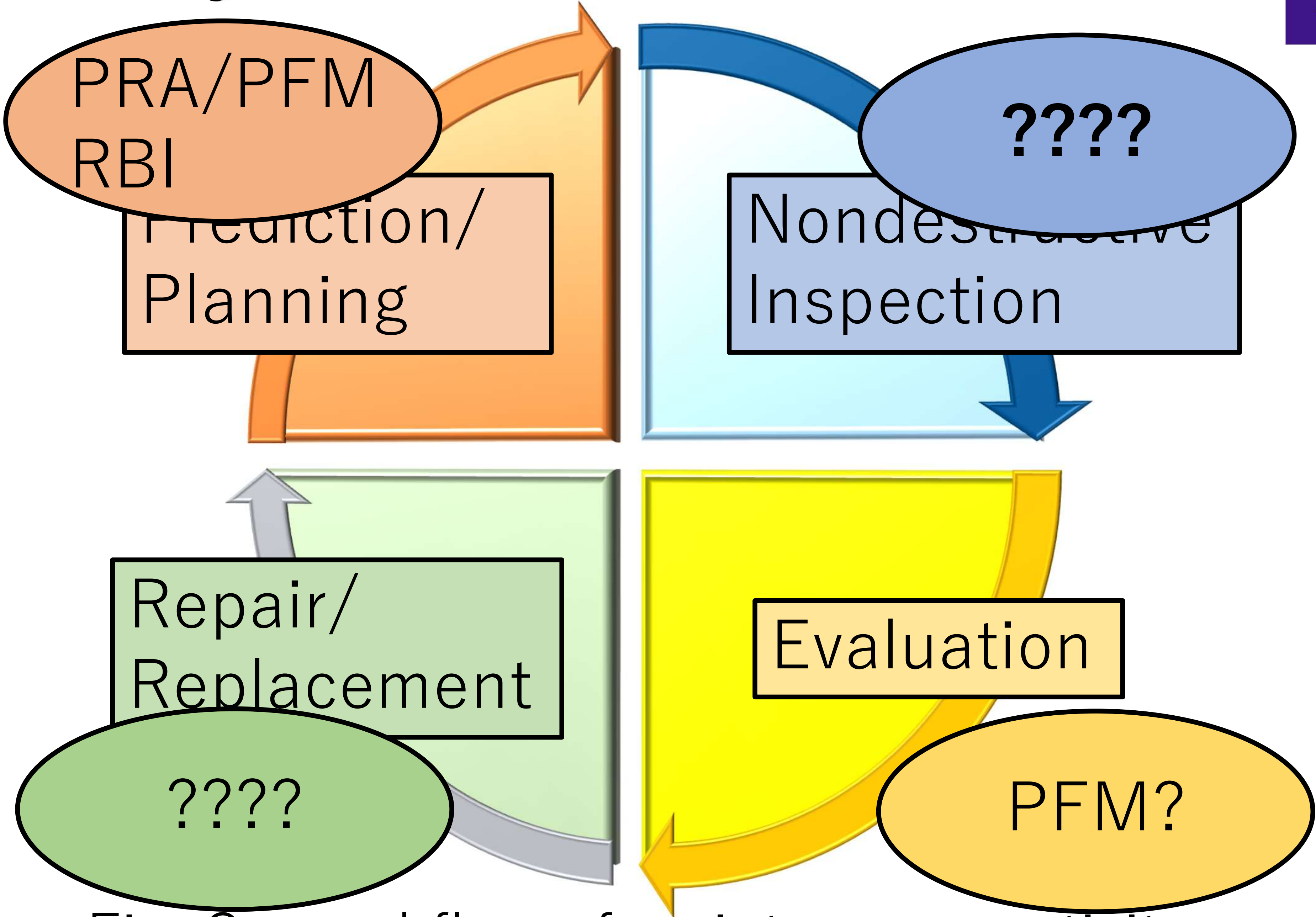


Fig. General flow of maintenance activity

1. Background and motivation

It is probable that risk-based management of infrastructure would be significantly enhanced, if

<u>the uncertainty of nondestructive evaluation is well quantified,</u>

but the number of such studies is quite limited.

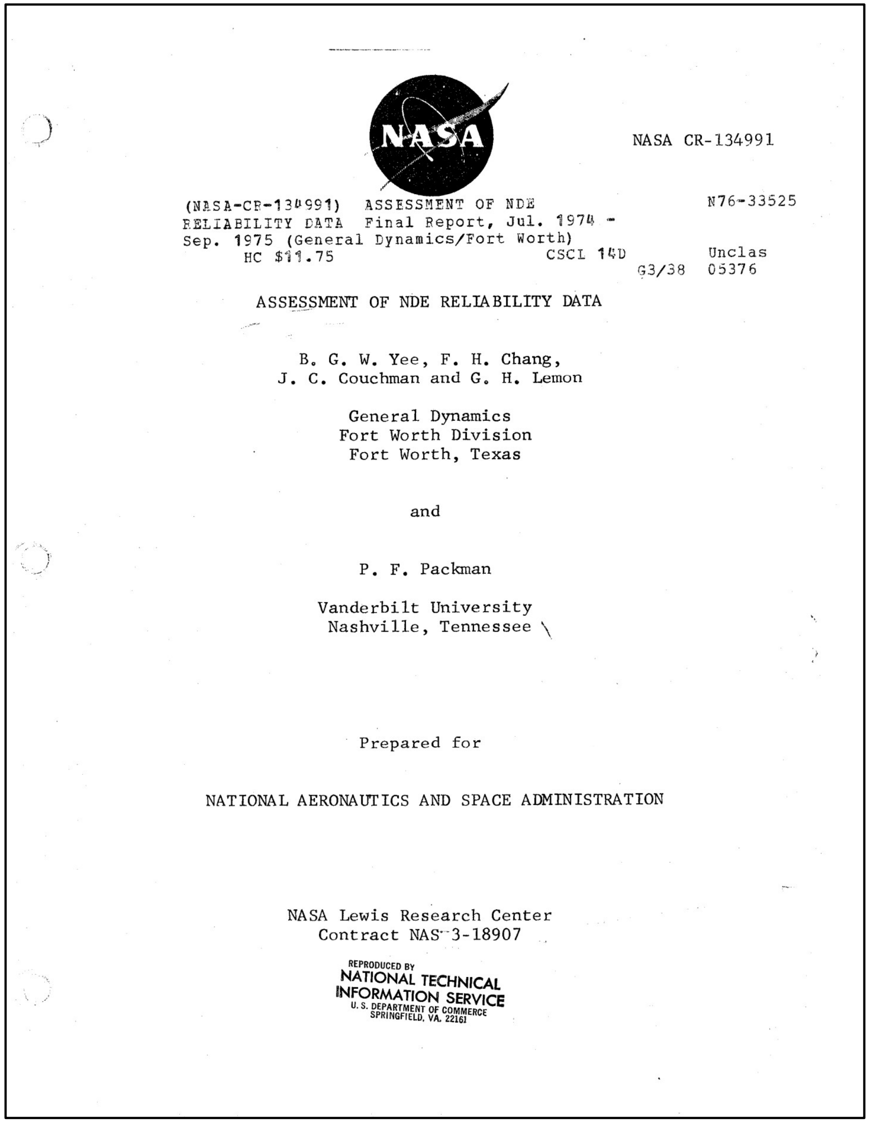
2 (1) Uncertainty of flaw detection

POD (Probability of Detection)

No need to detect all flaws.

2（1） Uncertainty of flaw detection

early POD studies (70's)



NASA CR-134991

(NASA-CR-134991) ASSESSMENT OF NDE RELIABILITY DATA Final Report, Jul. 1974 - Sep. 1975 (General Dynamics/Fort Worth) HC $11.75 CSCL 14D

N76-33525

Unclas G3/38 05376

ASSESSMENT OF NDE RELIABILITY DATA

B. G. W. Yee, F. H. Chang,
J. C. Couchman and G. H. Lemon

General Dynamics
Fort Worth Division
Fort Worth, Texas

and

P. F. Packman

Vanderbilt University
Nashville, Tennessee

Prepared for

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NASA Lewis Research Center
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NATIONAL TECHNICAL INFORMATION SERVICE
U. S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA. 22161

BGW. Yee et al., Assessment of NDE Reliability Data, NASA-CR-134991 (1976)

Directly, the probability of detection is given as

$$p = \frac{n}{N}$$

n ← detected

N ← number of flaws

The lower confidence interval of this probability can be calculated given as

$$1 - 0.95 = \sum_{i=n}^{N} \binom{N}{i} p_l^i (1 - p_l)^{N-i}$$

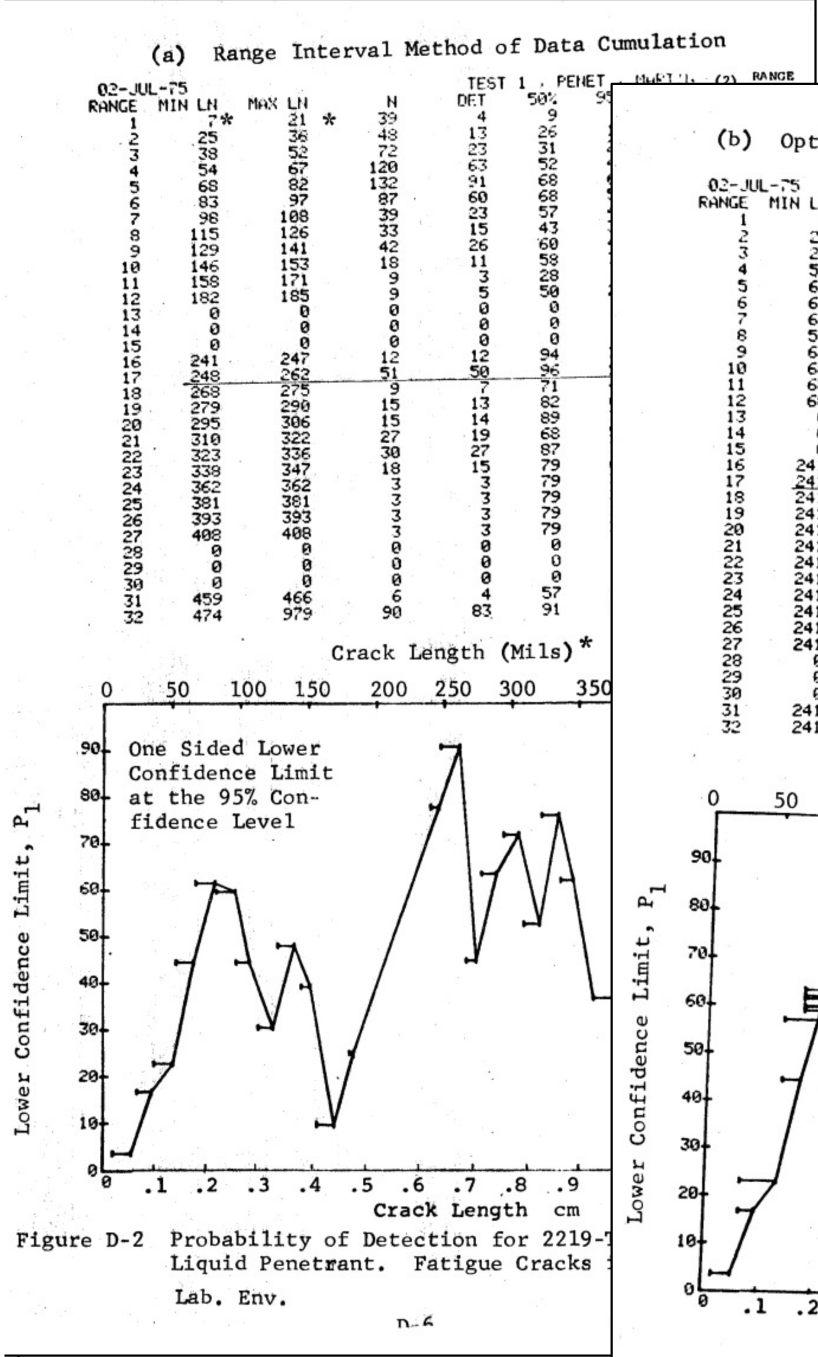


Figure D-2 Probability of Detection for 2219-T Liquid Penetrant. Fatigue Cracks Lab. Env.

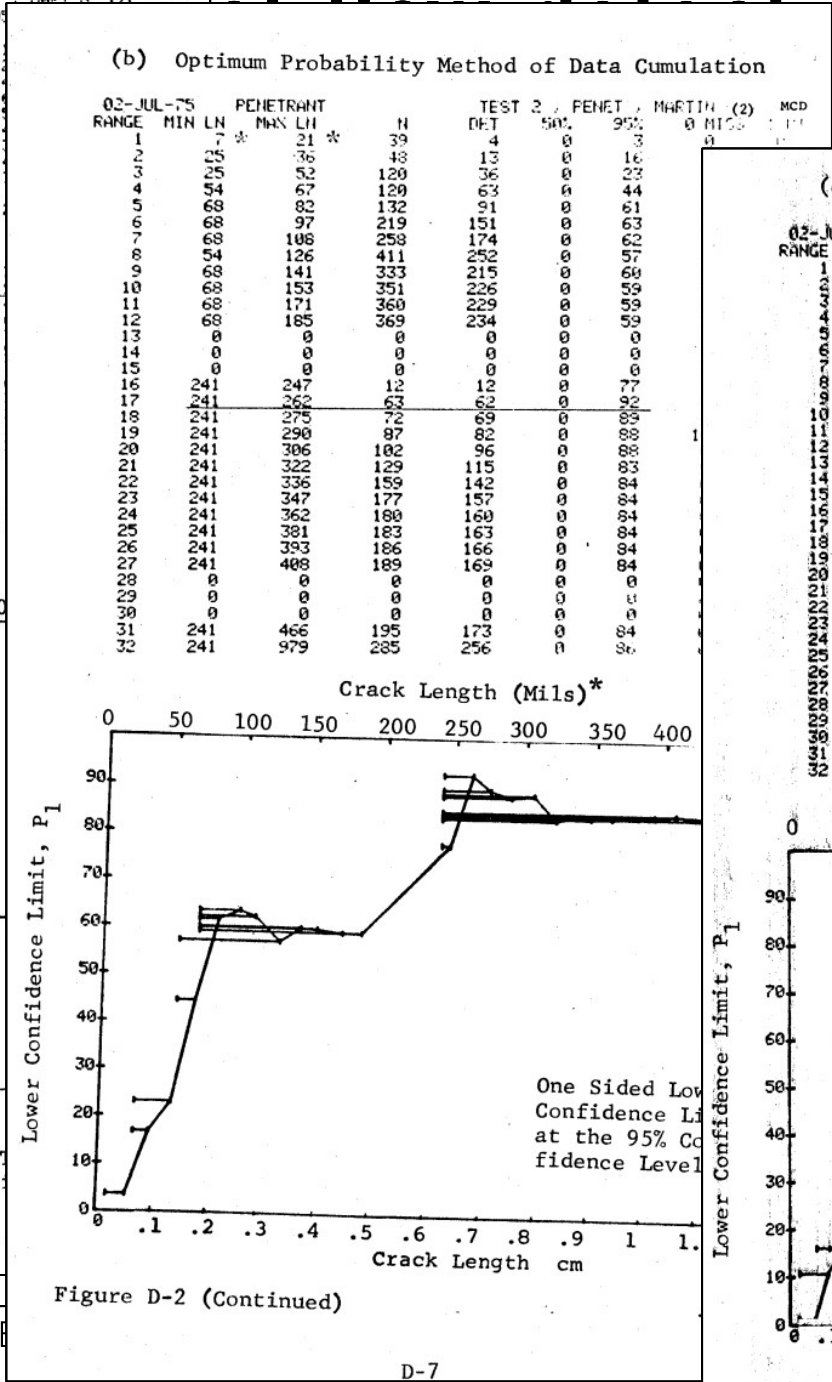


Figure D-2 (Continued)

D-7

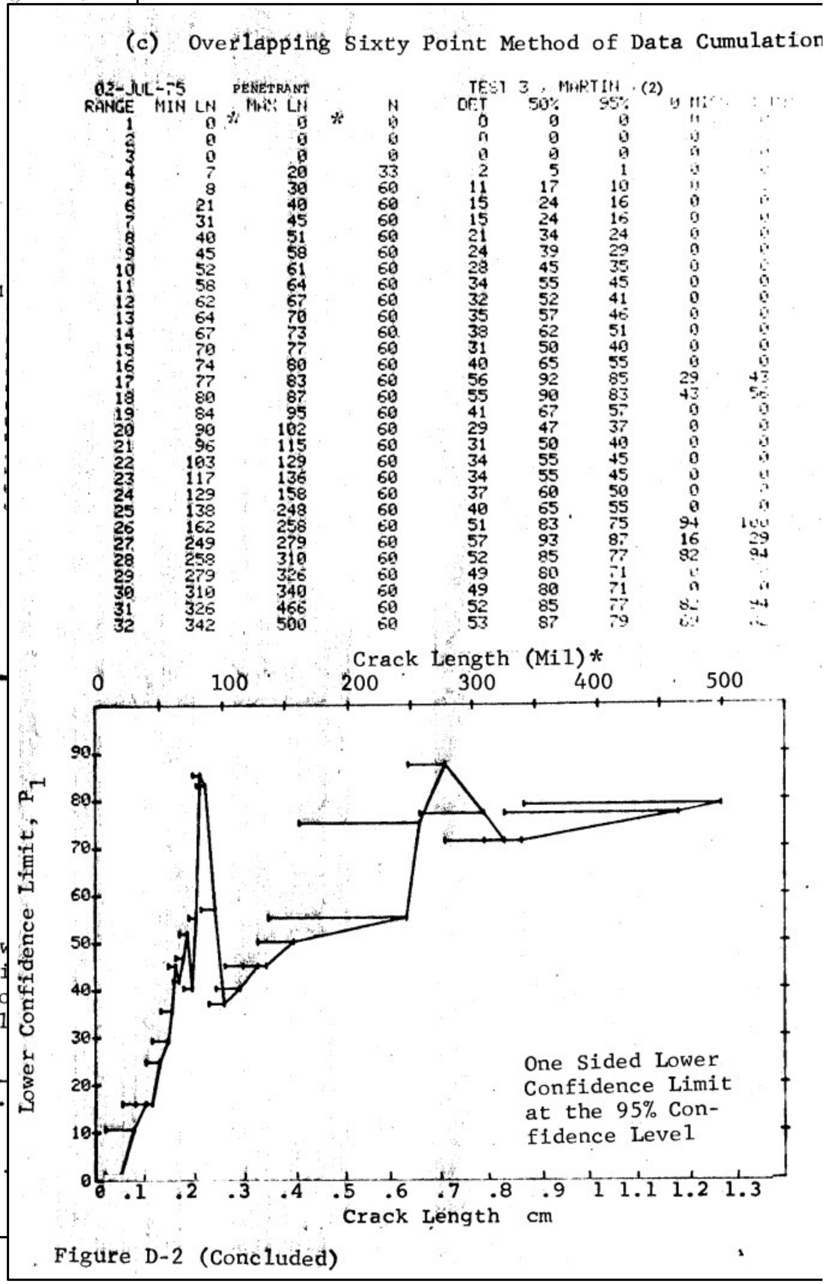


Figure D-2 (Concluded)

BGW. Yee et al., Assessment of NDE
NASA-CR-134991 (1976)

2 (1) Uncertainty of flaw detection

More recent POD studies

Results obtained by the approach based on the binomial distribution, which used in the early study, significantly depend on how to categorize flaws.

↓

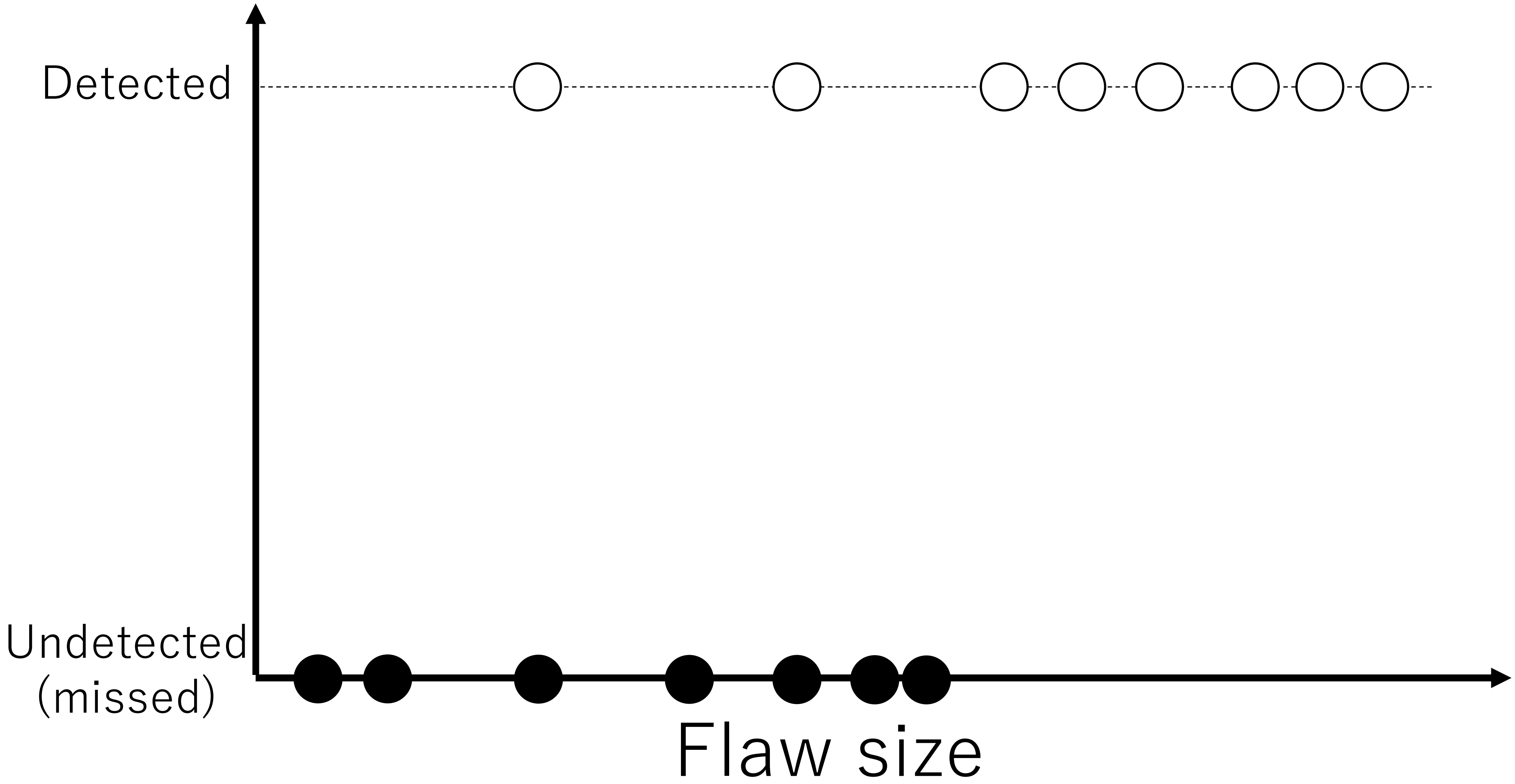
Estimating the parameters of a function that represents a probability of detection

- Binary data (detected or undetected)
  →Hit/Miss approach
- Signal amplitude > Threshold?
  →$\hat{a}$-a approach

2（1） Uncertainty of flaw detection

Recent POD: Hit/Miss approach

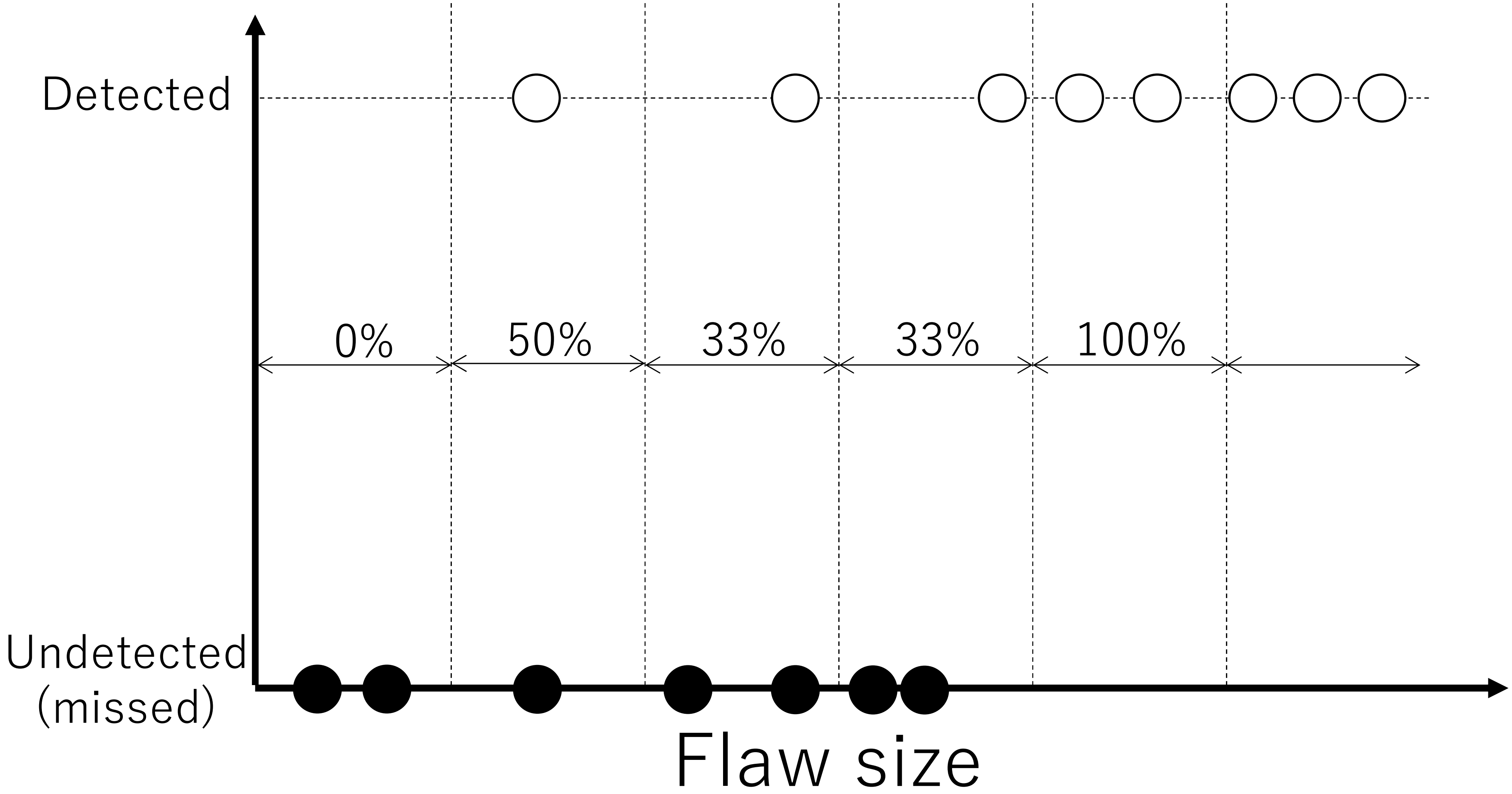
[For binary data]



2 (1) Uncertainty of flaw detection

Recent POD: Hit/Miss approach

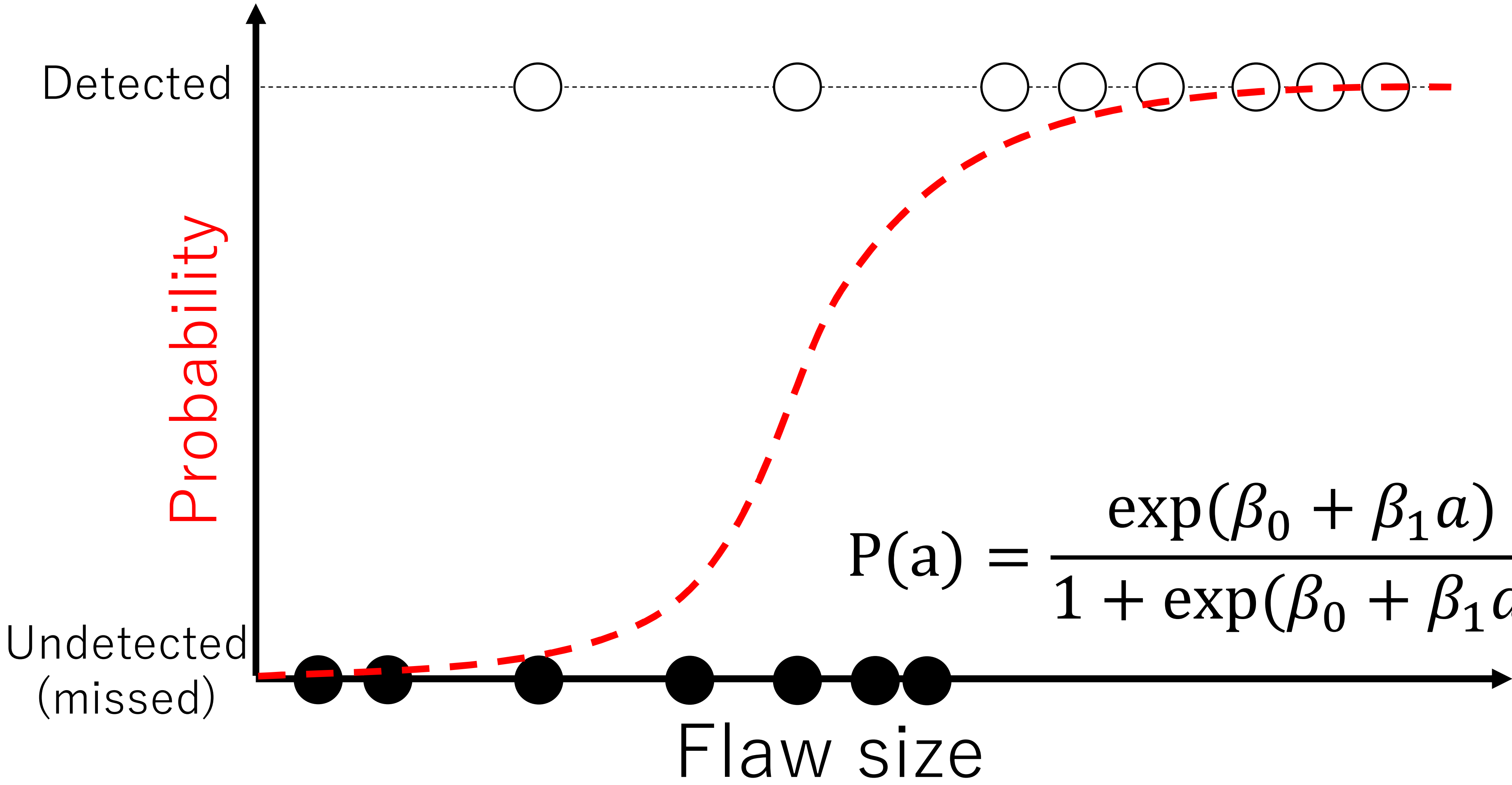
[The early approach]



2（1） Uncertainty of flaw detection

Recent POD: Hit/Miss approach

[Estimating the parameters of a function]

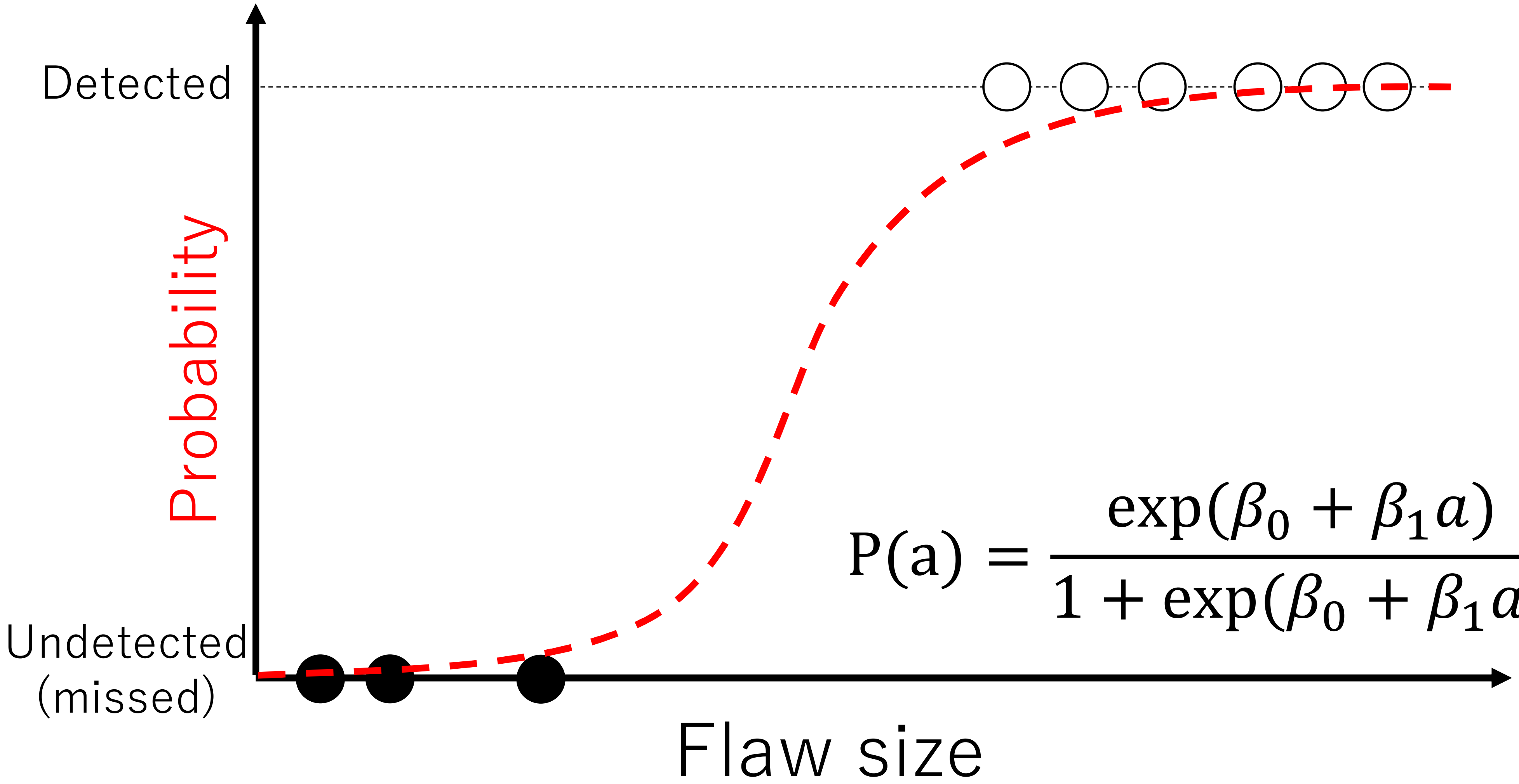


$$P(a) = \frac{\exp(\beta_0 + \beta_1 a)}{1 + \exp(\beta_0 + \beta_1 a)}$$

2（1） Uncertainty of flaw detection

Recent POD: Hit/Miss approach

[Problem]



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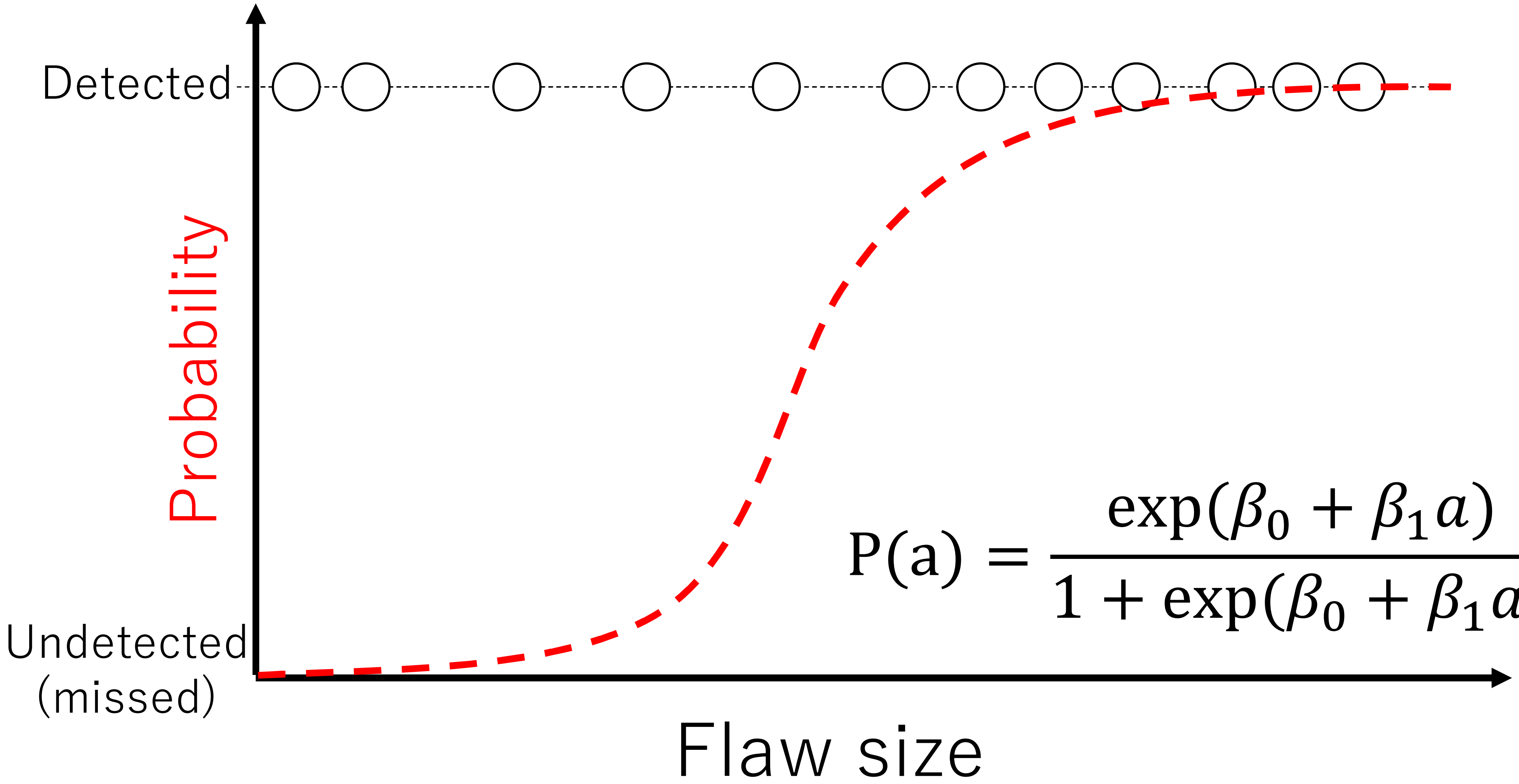
2 (1) Uncertainty of flaw detection

Recent POD: Hit/Miss approach

[Problem]

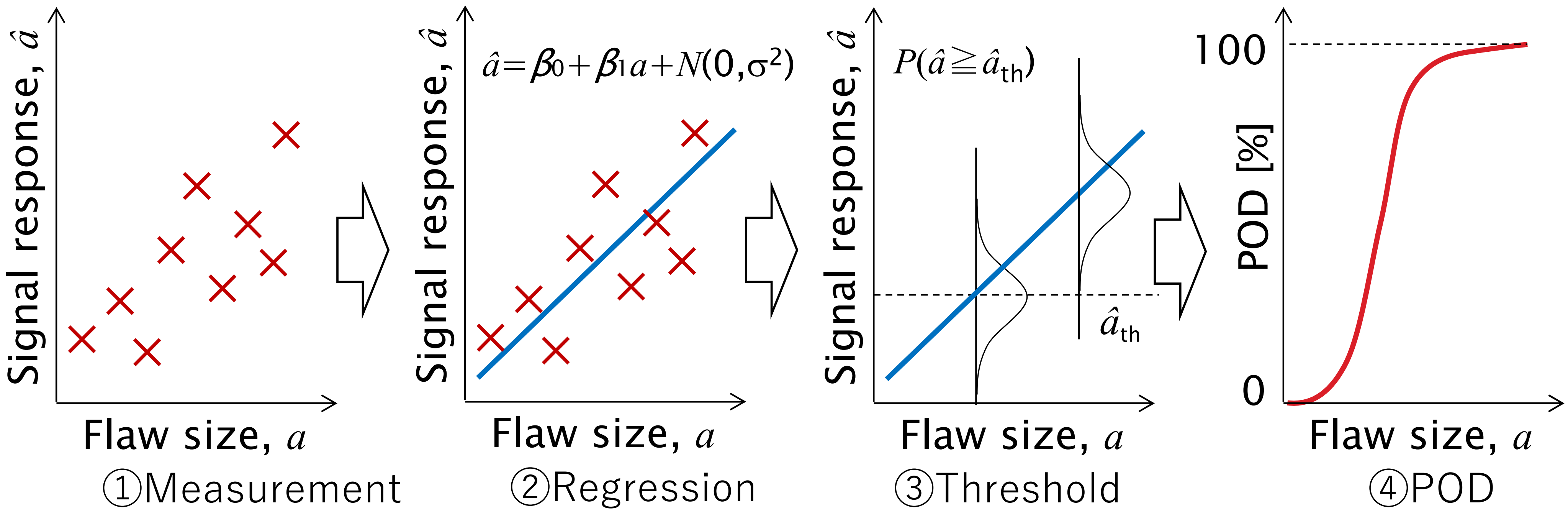


$$P(a) = \frac{\exp(\beta_0 + \beta_1 a)}{1 + \exp(\beta_0 + \beta_1 a)}$$

2（1） Uncertainty of flaw detection

Recent POD: $\hat{a}$-a approach

[For “signal amplitude” data]



- Applicable to the results where the results of measurements are given as numeric data (not binary).
- Capable of considering the effect of the decision threshold

2 (1) Uncertainty of flaw detection

Recent POD: $\hat{a}$-a approach

[For

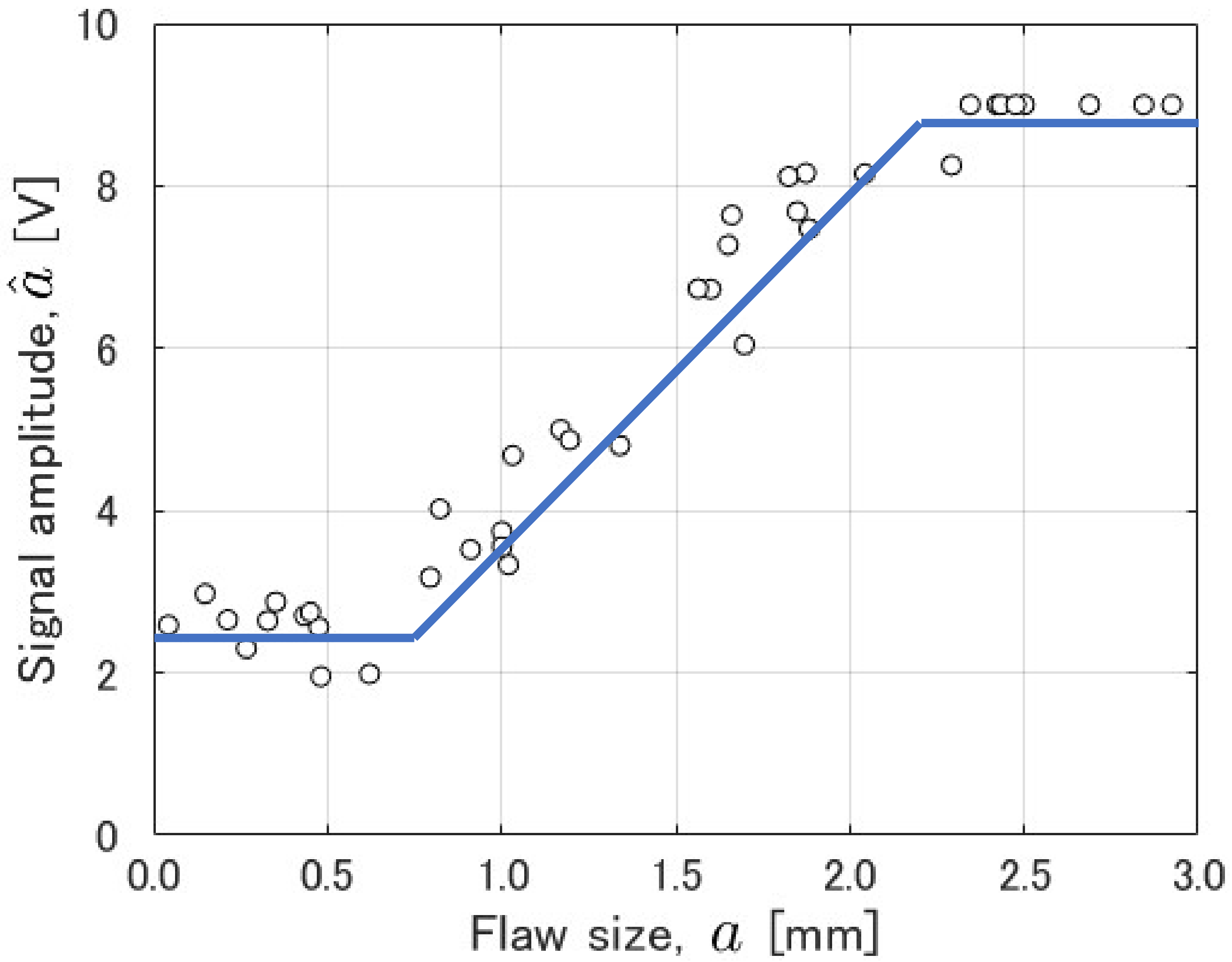
Signal response, $\hat{a}$

Fl

e, a

D

).

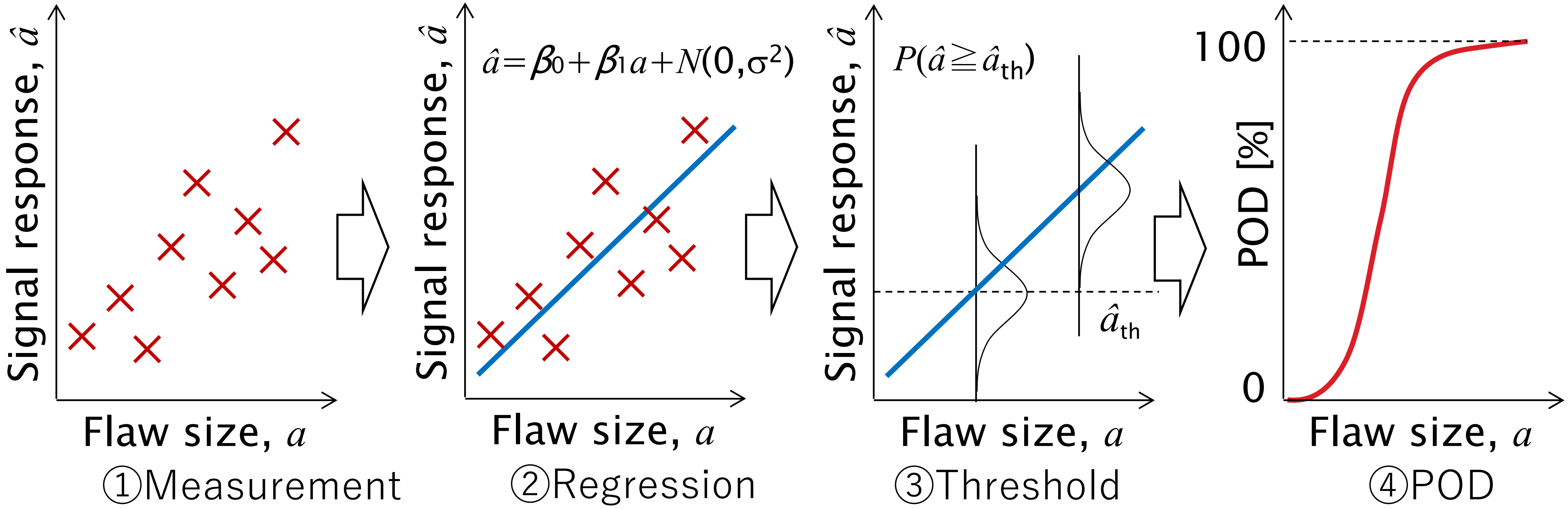


$$L=\prod_{i=1}^{N_1}\Phi\left(\frac{\hat{a}_l-(\beta_0+\beta_1 a_i)}{\sigma}\right)\times\prod_{i=N_1+1}^{N_1+N_2}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{\hat{a}_i-(\beta_0+\beta_1 a_i)}{\sigma}\right)\times\prod_{i=N_1+N_2+1}^{N_1+N_2+N_3}\Phi\left(\frac{(\beta_0+\beta_1 a_i)-\hat{a}_r}{\sigma}\right)$$

2 (1) Uncertainty of flaw detection

Recent POD: $\hat{a}$-a approach

[For “signal amplitude” data]



Problems

- Simple linear regression model
- Constant variance is necessary
- Many experimental signals (&samples) needed
- A single parameter to characterize a flaw

2（1） Uncertainty of flaw detection

Our approach for reconstructing the $\hat{a}$-a approach

■ Simple regression model
→Not closed-form

■ Constant variance
→Variance depending on flaw size

■ Many experimental signals (&samples)
→Combinational use of measurements and simulations

■ A single parameter to characterize flaw
→Multiple flaw parameters

2（1） Uncertainty of flaw detection

① detecting surface cracks by eddy current testing

1. Regression analysis based on the combinational use of measured and simulated signals

Measured — Normal distribution — Simulated

$$V(d,l) = N(\mu_1,\sigma_1^2) \times V^{sim}(d,l) + N(\mu_2,\sigma_2^2)$$

Depth — Length — Normal distribution

2. Maximum likelihood analysis for estimating the parameters

$$\ln L = \sum_{i=1}^{M_l} \ln \Phi\left(\frac{V_l - \left(\mu_1 V^{sim}(d_i,l_i) + \mu_2\right)}{\sqrt{V^{sim}(d_i,l_i)^2\sigma_1^2 + \sigma_2^2}}\right) - \frac{1}{2}\sum_{i=M_l+1}^{M-Mr}\left[\ln\left\{2\pi\left(V^{sim}(d_i,l_i)^2\sigma_1^2 + \sigma_2^2\right)\right\} + \frac{\left\{V_i - \left(\mu_1 V^{sim}(d_i,l_i) + \mu_2\right)\right\}^2}{V^{sim}(d_i,l_i)^2\sigma_1^2 + \sigma_2^2}\right]$$
$$+ \sum_{i=M-M_r+1}^{M} \ln\left(1 - \Phi\left(\frac{V_r - \left(\mu_1 V^{sim}(d_i,l_i) + \mu_2\right)}{\sqrt{V^{sim}(d_i,l_i)^2\sigma_1^2 + \sigma_2^2}}\right)\right)$$

3. Probability of detection given as the probability that measured signal exceeds a threshold

$$POD(d,l) = \Phi\left(\frac{\left(\mu_1 V^{sim}(d,l) + \mu_2\right) - V_{th}}{\sqrt{V^{sim}(d,l)^2\sigma_1^2 + \sigma_2^2}}\right)$$

- N. Yusa et al, Demonstration of probability of detection taking consideration of both the length and the depth of a flaw explicitly, NDT&E International 81 (2016), 1-8.

2（1） Uncertainty of flaw detection

① detecting surface cracks by eddy current testing

Two-dimensional POD

90% POD + Confidence interval

- N. Yusa et al, Demonstration of probability of detection taking consideration of both the length and the depth of a flaw explicitly, NDT&E International 81 (2016), 1-8.

2 (1) Uncertainty of flaw detection

② effect of the distance between scanning lines

- 36 fatigue cracks on type 316L SS
- pluspoint probe, 100kHz
- one flaw parameter model
- scanning line runs parallel to a crack

Length & depth of the fatigue cracks

Depth vs FWHM of signal distribution
(when probe runs perpendicular to a fatigue crack)

- N. Yusa et al., Probabilistic evaluation the area of coverage of a probe used for eddy current non-destructive inspections, International Journal of Applied Electromagnetics and Mechanics 64 (2020), 11-18.

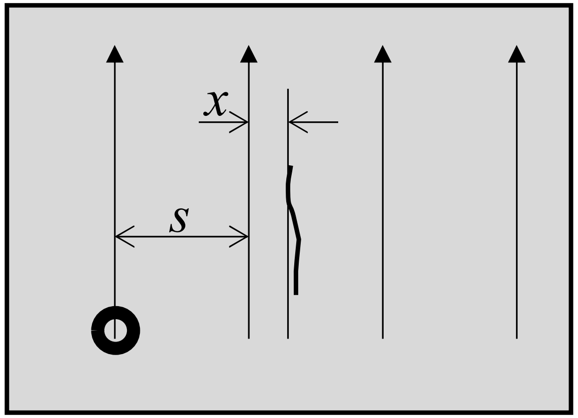
2（1） Uncertainty of flaw detection

② effect of the distance between scanning lines

Then, evaluate

$$\frac{1}{s}\int_{-s/2}^{s/2}\int_{\sqrt{V_{th}}}^{\infty}\underbrace{\sqrt{F(x)}}_{\text{distance}}\underbrace{P\left(V^{1/2};d\right)}_{\text{signal}}dV^{1/2}\,dx$$

$$x = U\left(-\frac{s}{2},\frac{s}{2}\right)$$



by the Monte-Carlo method to evaluate signal.

s=3 mm　　s=5 mm　　s=7 mm

Probability of detection with variable distances

- N. Yusa et al., Probabilistic evaluation the area of coverage of a probe used for eddy current non-destructive inspections, International Journal of Applied Electromagnetics and Mechanics 64 (2020), 11-18.

2（1） Uncertainty of flaw detection

③ effect of sensor placing in wall thinning monitoring

The POD is given as the probability

$$|\boldsymbol{B}^{\boldsymbol{exp}}(l, t_r, \theta, s_A, s_C)| > B_{th}$$

[Assumption1]

Fig. The dimensions and sensor placements

$$B_{i,j}^{exp}(l, \theta, t_r) = c_1 B_{i,j}^{sim}(l, \theta, t_r) + c_2 + N(0, \sigma_e^2)$$

$$\sigma_e = c_3 B_{i,j}^{sim} + c_4$$

[Assumption2]

$$x_A \sim Uniform\left(-\frac{s_A}{2}, \frac{s_A}{2}\right) \ \& \ x_C \sim Uniform\left(-\frac{s_C}{2}, \frac{s_C}{2}\right)$$

- The four parameters were estimated by comparing experimental and numerical signals due to 27 ($i = 1, 2, ..., 27$) samples.
- The effect of x_A and x_C were evaluated by Monte-Carlo simulations (N=1,000,000).
- The confidence interval of POD was calculated by the bootstrap method (1,000 samples).

- H. Song, N. Yusa, A probability of detection model for a sensor-based monitoring method against local wall thinning, International Journal of Applied Electromagnetics and Mechanics 71 (2023), S29-S37.

2 (1) Uncertainty of flaw detection

③ effect of sensor placing in wall thinning monitoring

Fig. POD contour when S_A=50 mm and S_C=90°

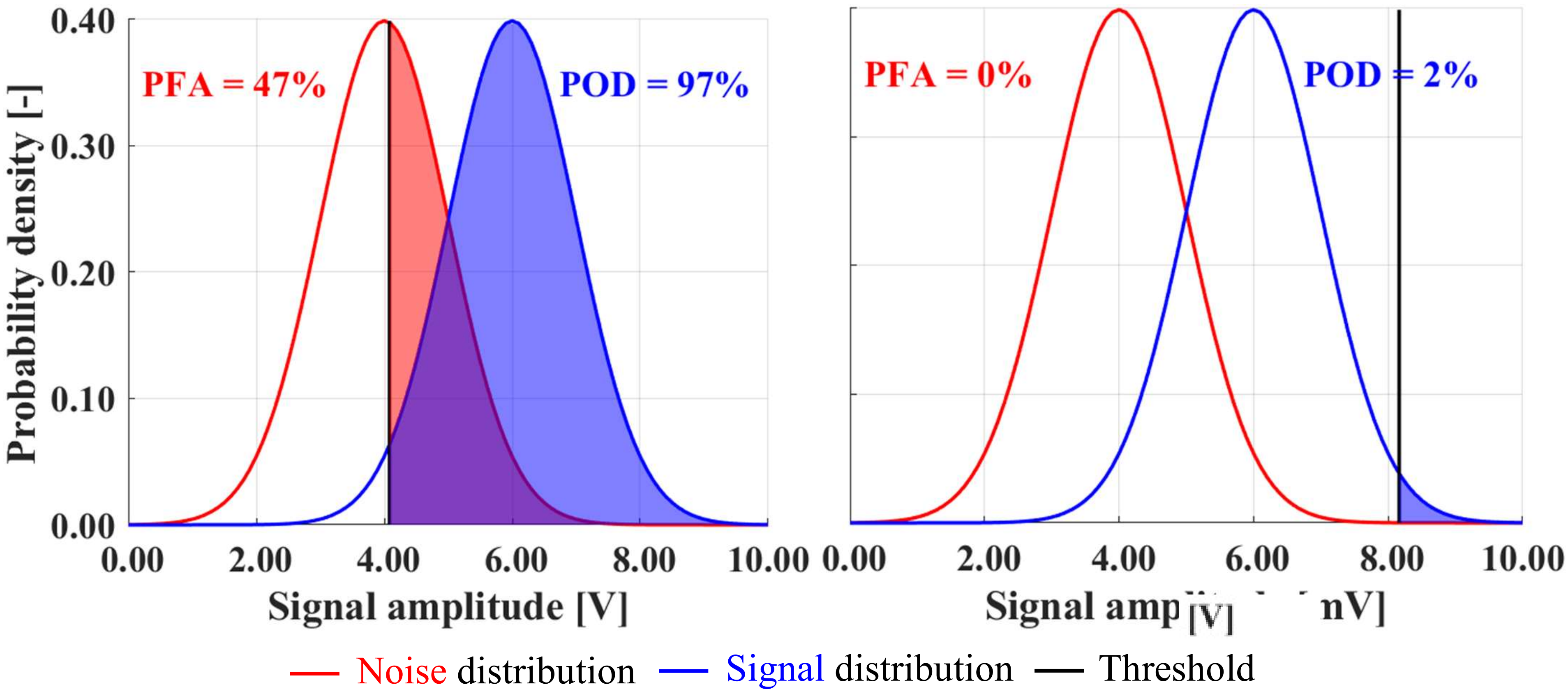
Fig. POD contour when S_A=50 mm and S_C=60°

- H. Song, N. Yusa, A probability of detection model for a sensor-based monitoring method against local wall thinning, International Journal of Applied Electromagnetics and Mechanics 71 (2023), S29-S37.

2（1）Uncertainty of flaw detection

④ from POD to ROC

A lower a_{th} leads not only higher POD but also a higher PFA



→Necessity to consider both POD and PFA

- F. Yu et al, Receiver operating characteristic analysis for evaluating a proper experimental condition of eddy current tests under a low signal-to-noise ratio, International Journal of Applied Electromagnetics and Mechanics 71 (2023), S179-S189.

2（1） Uncertainty of flaw detection

④ from POD to ROC

- F. Yu et al, Receiver operating characteristic analysis for evaluating a proper experimental condition of eddy current tests under a low signal-to-noise ratio, International Journal of Applied Electromagnetics and Mechanics 71 (2023), S179-S189.

2 (2) Uncertainty of flaw evaluation

Evaluating (sizing) a flaw is an inverse problem

$$y = f(x)$$: forward problem

$$x = f^{-1}(y)$$: inverse problem

ill-posedness of inverse problems (from NDT viewpoint)

- there would be no x that gives y, (necessity of proper flaw modeling);
- two x provides the same y;
- small change in y would lead to a large change in x (small noise would lead to a large error).

2 (2) Uncertainty of flaw evaluation

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2 (2) Uncertainty of flaw evaluation

An earlier study

- N. Yusa et al., Caution when applying eddy current inversion to stress corrosion cracking, Nuclear Engineering and Design 236 (2006), 211-221.

2 (2) Uncertainty of flaw evaluation

An earlier study ~ huge error!!

The estimated profile differed significantly from the true one, although the signal was well reproduced.

→ Necessity to evaluate the uncertainty

- N. Yusa et al., Caution when applying eddy current inversion to stress corrosion cracking, Nuclear Engineering and Design 236 (2006), 211-221.

2 (2) Uncertainty of flaw evaluation

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2 (2) Uncertainty of flaw evaluation

Evaluating possible error range ~ an earlier approach

Calculating all possible (flaw profile)-(signal) combinations.

(a) Flaw model1

(b) Flaw model2

Model (flaw on a t10 plate)

Possible error caused by 10% signal difference

■ Small error would lead to a large error in flaw evaluation

→Point estimation is insufficient

- N. Yusa et al, Numerical evaluation of the ill-posedness of eddy current problems to size real cracks, NDT&E International 40 (2007), 185-191.
- N. Yusa, H. Hashizume, Numerical investigation of the ability of eddy current testing to size surface breaking cracks, Nondestructive Testing and Evaluation 32 (2017), 50-58.

2 (2) Uncertainty of flaw evaluation

More quantitative approach

The Bayes' theorem states

posterior distribution — likelihood — prior distribution

$$P(\boldsymbol{X}|\boldsymbol{V}) = \frac{P(\boldsymbol{V}|\boldsymbol{X})P(\boldsymbol{X})}{P(\boldsymbol{V})}$$

flaw profile — signal

from POD analysis

Thus,

$$P(\boldsymbol{X}|\boldsymbol{V}_1, \boldsymbol{V}_2, \cdots, \boldsymbol{V}_{\mathrm{N}}) = \frac{P(\boldsymbol{X}) \prod_{i=1}^{\mathrm{N}} P(\boldsymbol{V}_i|\boldsymbol{X})}{\int P(\boldsymbol{X}) \prod_{i=1}^{\mathrm{N}} P(\boldsymbol{V}_i|\boldsymbol{X})\, dX}$$

- C. Cal et al., Metamodel-based Markov-Chain-Monte-Carlo parameter inversion applied in eddy current flaw characterization, NDT&E International 99 (2018), 13-22.
- T. Tomizawa and N. Yusa, Bayesian data fusion of eddy current testing for flaw characterization with uncertainty evaluation, NDT&E International (under review)

3. Summary

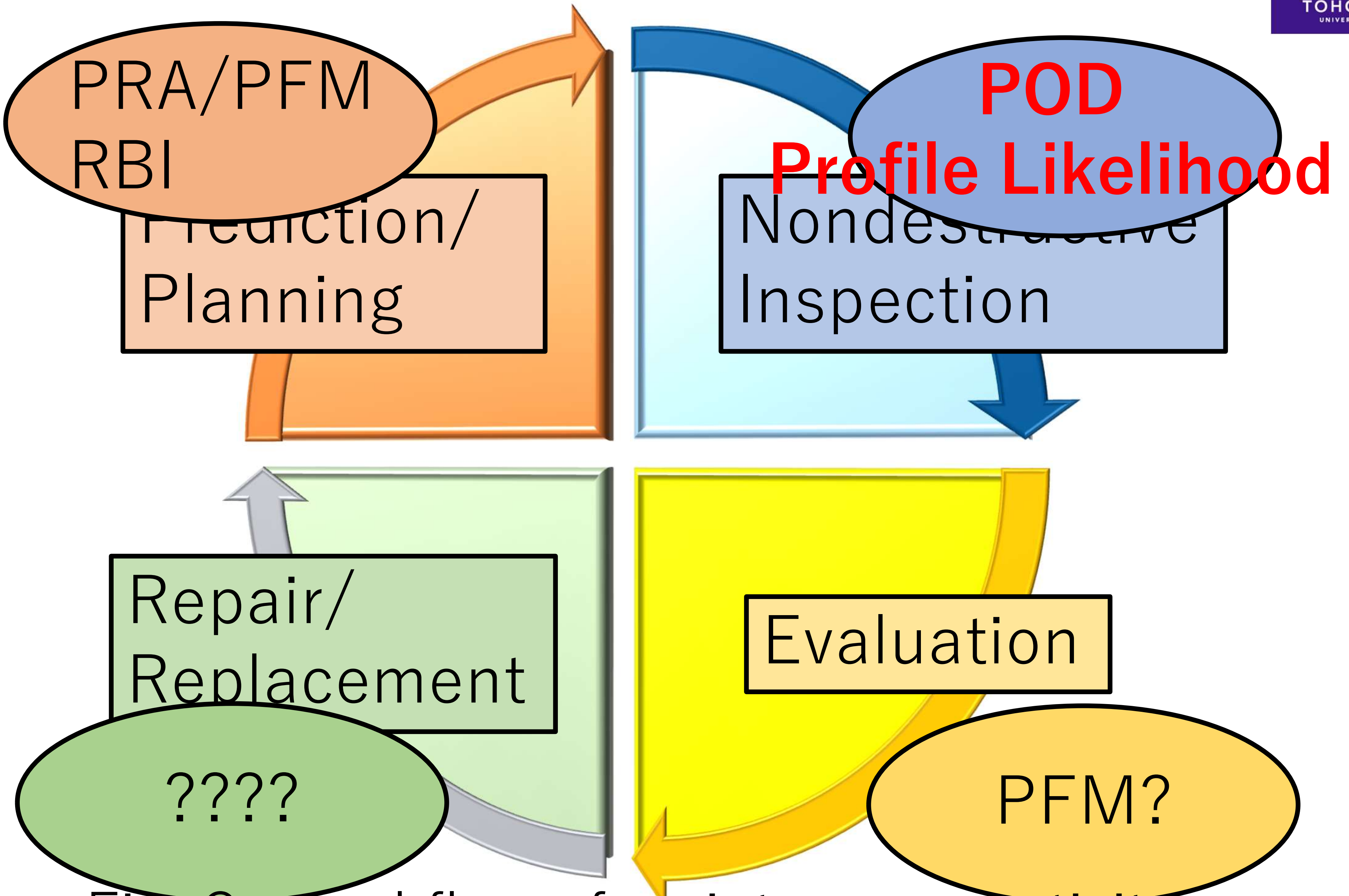

Fig. General flow of maintenance activity

Thank you for your attention.